

Math 308 H, Autumn 2007
Midterm 1
October 17, 2007

Name: _____

1	15	
2	15	
3	15	
4	15	
Total	60	

- You may use a scientific calculator during this examination. Other electronic devices are not allowed, and should be turned off for the duration of the exam.
- You may use one side of 8.5 by 11 inch page of notes. It must be handwritten.
- Show all work for full credit.
- You have 80 minutes to complete the exam.

1. Let $A = \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 1 & 2 & -1 & 1 & 3 \\ 1 & 4 & 1 & 2 & -1 \end{bmatrix}$.

(a) (10 pts) Find all solutions to $Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Give your answer in vector form.

(b) (5 pts) Write A_5 , the fifth column of A , as a linear combination of the other columns of A .

2. (a) (5 pts) Let A and B be (4×3) matrices such that $A \begin{bmatrix} -2 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ and $B \begin{bmatrix} -2 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$. What can you say about the rank of the matrix $(2A - B)$? Explain your answer.

(b) (5 pts) Let A be a (4×4) matrix with $\text{rank}(A) = 3$. What can you say about the number of solutions to the matrix equation $Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$? Explain your answer.

(c) (5 pts) Suppose that the set $\{v_1, v_2, v_3\}$ is linearly dependent. Must the set $S = \{v_1, v_2\}$ also be linearly dependent? What about the set $T = \{v_1, v_2, v_3, v_4\}$? Explain your answers.

3. Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$

Find (2×2) matrices C and D such that $AC = B$ and $BD = A$. What is the relationship between C and D ?

4. (15 pts) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$. Let A be a (4×3) matrix such that

$$Av_1 = \begin{bmatrix} -2 \\ 1 \\ 3 \\ 5 \end{bmatrix}, Av_2 = \begin{bmatrix} 1 \\ -3 \\ 4 \\ -1 \end{bmatrix}, \text{ and } Av_3 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ -4 \end{bmatrix}. \text{ Find such a matrix } A.$$