Midterm 2 Practice Problems

Question 1

This question concerns the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 3 & 7 \\ 2 & 4 & 2 & 7 & 18 \end{bmatrix}$.

1. Find a basis for $\mathcal{N}(A)$.
2. Find a basis for $\mathcal{R}(A)$
3. Find a basis for the row space of A.
4. Suppose that B is another (2×5) matrix. Assume that you are told that A and B are row-equivalent. True or False: $\mathcal{N}(A) = \mathcal{N}(B)$. Explain your answer briefly.
5. As in part (d), suppose that B is a (2×5) matrix which is row-equivalent to A. True or False: $\mathcal{R}(A) = \mathcal{R}(B)$. Explain your answer briefly.

Question 2

1. Give an example of a basis for $\mathbb{R}^3$.
2. Give an example of a spanning set for $\mathbb{R}^3$ which is NOT a basis for $\mathbb{R}^3$.
3. Give an example of a linearly independent set of vectors in $\mathbb{R}^4$ which is NOT a basis for $\mathbb{R}^4$.

Question 3

Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation, and that

$$T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, T\left(\begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$$

1. Find $T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$ and $T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)$.
2. Find a matrix A such that $T(x) = Ax$.

Question 4

In this question, suppose that C is an unspecified (3×3) matrix. Suppose that you are told that the matrix equations $Cx = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$ and $Cx = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$ are both solvable. Suppose that you are also told that $C \begin{bmatrix} 9 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Without knowing anything else about the matrix C, answer the following questions. Please explain your answers carefully. Your explanations should be brief, but convincing.

1. Describe the range of C as precisely as you can.
2. Describe the null space of C as precisely as you can.
3. How many solutions does the matrix equation $Cx = \begin{bmatrix} 8 \\ 11 \\ 15 \end{bmatrix}$ have?

Question 5

Find the best least-squares quadratic fit (of the form $y = at^2 + bt + c$) for the data points

t	-2	-1	1	2
y	4	0	2	4

Question 6

Answer True or False. If the answer is True, briefly explain why. If it is False, give a counter example.

1. If W is a subspace, then W has exactly one basis.
2. If W is a subspace and dim(W)=p, then W has exactly p elements.
3. If B is a basis for $\mathbb{R}^n$, and W is a subspace of $\mathbb{R}^n$, then some subset of B is a basis for W.
4. If W is a subspace of $\mathbb{R}^n$, and dim(W)=n, then $W = \mathbb{R}^n$.
5. If W is a subspace of $\mathbb{R}^n$, then $V = \{x \in \mathbb{R}^n \ : \ x \notin W\}$ is also a subspace.