

Print your name: _____

Problem	Points	Score
1	4	
2	10	
3	10	
4	10	
5	11	
Take home	15	
Total	60	

Check one:

_____ I would like my exam placed in a box outside Nathan's office from Saturday morning until Tuesday morning.

_____ Please do not put my exam in a box outside Nathan's office.

Regardless of what you answer above, you can pick up your exam directly from me on Tuesday between 9 and 10 am, or any time during Autumn quarter. I will shred all remaining exams at the end of the year 2011.

Instructions:

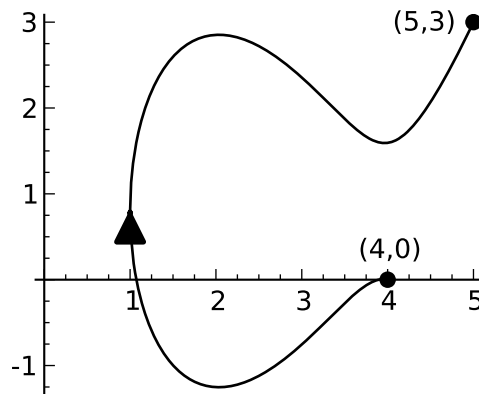
- Write complete solutions.
- | |
|-----------------------|
| Box your final answer |
|-----------------------|

 when applicable.
- Write on the backs of the pages if you need more room.
- Do not use any electronic device other than a non-graphing calculator.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on any part of this exam, including the take home problem.

Signature: _____

1. (4 points) Let $f = x + xy + y$, and let C be the curve below, with endpoints $(4, 0)$ and $(5, 3)$, and oriented in the clockwise-ish direction.



Determine $\int_C \nabla f \cdot d\mathbf{r}$.

2. (10 points) Let C_1 be the spiral parametrized by $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$, $0 \leq t \leq 6\pi$. Let C_2 be the line segment from $(1, 0, 6\pi)$ to $(1, 0, 0)$. Let C be C_1 followed by C_2 . Determine

$$\int_C \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, z \right\rangle \cdot d\mathbf{r}.$$

3. (10 points) Let C be the path that goes in a straight line from $(1, 0, 0)$ to $(0, -1, 0)$ to $(0, 0, 1)$ and back to $(1, 0, 0)$. Use Stokes' Theorem to set up a double integral that computes

$$\int_C \langle xyz, x + y, x + z \rangle \cdot d\mathbf{r}.$$

Do not evaluate. Your answer should have two variables only and no vectors, looking something like this: $\int_{-} \int_{-} \text{---} dx dy$.

4. (10 points) Let E be the region above the plane $y + z = -6$, below the plane $x + z = 6$, and inside the cylinder $x^2 + y^2 = 9$. Let S be the boundary of E (the sides of the cylinder + the ellipse at the top + the ellipse at the bottom) with the positive (outward) orientation. Calculate

$$\iint_S \langle x^3, z^3, 3y^2z \rangle \cdot d\mathbf{S}.$$

5. Let $\mathbf{F} = \frac{\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{3/2}}$, the vector field from the take home problem. If you did the problem right, you noticed that $\text{curl } \mathbf{F} = \mathbf{0}$ and $\text{div } \mathbf{F} = 0$ everywhere except for at the origin, where it isn't defined. Also, the integral of the vector field over a sphere of any radius is 4π .

(a) (3 points) Use the Divergence Theorem to explain why $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$ if S is the sphere of radius 1 centered at $(2, 2, 2)$.

(b) (5 points) Find a function f defined everywhere except at the origin so that $\nabla f = \mathbf{F}$, or explain why no such function exists.

(c) (3 points) Explain why there is no vector field \mathbf{G} such that $\mathbf{F} = \text{curl } \mathbf{G}$.