### Math 407A: Linear Optimization

#### Lecture 8: Initialization and the Two Phase Simplex Algorithm

Math Dept, University of Washington

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We now address the question of how to determine an initial feasible dictionary (tableau).

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$$\begin{aligned} \mathcal{P} \quad & \text{maximize} \quad c^{\mathsf{T}} x \\ & \text{subject tp} \quad A x \leq b, \quad 0 \leq x. \end{aligned}$$

Consider an auxiliary LP of the form

where  $\mathbf{1} \in \mathbb{R}^m$  is the vector of all ones.

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where  $\mathbf{1} \in \mathbb{R}^m$  is the vector of all ones. The i<sup>th</sup> row of the system of inequalities  $Ax - x_0 \mathbf{1} \leq b$  is

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \leq b_i + x_0$$
.

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In block matrix form we write

$$\begin{bmatrix} -1 & A \end{bmatrix} \begin{pmatrix} x_0 \\ x \end{bmatrix} \leq b$$
.

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If the optimal value in the auxiliary problem is zero, then at the optimal solution  $(\tilde{x}_0, \tilde{x})$  we have  $\tilde{x}_0 = 0$ .

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Plugging into  $Ax - x_0 \mathbf{1} \leq b$ , we get  $A\tilde{x} \leq b$ , i.e.  $\tilde{x}$  is feasible for  $\mathcal{P}$ .

 $egin{array}{ccc} \mathcal{Q} & \mbox{minimize} & x_0 \ & \mbox{subject to} & Ax - x_0 \mathbf{1} \leq b, & 0 \leq x_0, x \ . \end{array}$ 

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Plugging into  $Ax - x_0 \mathbf{1} \leq b$ , we get  $A\tilde{x} \leq b$ , i.e.  $\tilde{x}$  is feasible for  $\mathcal{P}$ .

On the other hand, if  $\hat{x}$  is feasible for  $\mathcal{P}$ , then  $(\hat{x}_0, \hat{x})$  with  $\hat{x}_0 = 0$  is feasible for  $\mathcal{Q}$ , so  $(\hat{x}_0, \hat{x})$  is optimal for  $\mathcal{Q}$ .

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$$\begin{array}{cccc} \mathcal{P} & \mbox{maximize} & c^{\mathsf{T}}x & \mathcal{Q} & \mbox{minimize} & x_0 \\ & \mbox{subject tp} & Ax \leq b, & \mbox{subject to} & Ax - x_0 \mathbf{1} \leq b, \\ & 0 \leq x & 0 \leq x_0, x \end{array}$$

•  $\mathcal{P}$  is feasible  $\Leftrightarrow$  the optimal value in  $\mathcal{Q}$  is zero.

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• 
$$\mathcal{P}$$
 is feasible  $\Leftrightarrow$  the optimal value in  $\mathcal{Q}$  is zero.

•  $\mathcal{P}$  is infeasible  $\Leftrightarrow$  the optimal value in  $\mathcal{Q}$  is positive.

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The auxiliary problem Q is also called the *Phase I* problem since solving it is the first phase of a two phase process of solving general LPs.

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In Phase I we solve the auxiliary problem to obtain an initial feasible tableau for  $\mathcal{P}$ , and in Phase II we solve the original LP starting with the feasible tableau provided in Phase I.

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# Initializing the Auxiliary Problem

 $\mathcal{Q}$  minimize  $x_0$ subject to  $Ax - x_0 \mathbf{1} \leq b$ ,  $0 \leq x_0, x$ .

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Problem: The initial dictionary for  $\mathcal{Q}$  is infeasible!

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 minimize  $x_0$   
subject to  $Ax - x_0 \mathbf{1} \leq b$ ,  $0 \leq x_0, x$ .

Problem: The initial dictionary for  $\mathcal{Q}$  is infeasible!

Solution: Set 
$$x_0 = -\min\{b_i : i = 0, ..., n\}$$
 with  $b_0 = 0$ ,  
then  $b + x_0 \mathbf{1} \ge 0$  since

$$\min\{b_i + x_0 : i = 1, \dots, m\} = \min\{b_i : i = 1, \dots, m\} + x_0$$
$$= \min\{b_i : i = 1, \dots, m\} - \min\{b_i : i = 0, \dots, m\} \ge 0.$$

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 $= \min\{b_i : i = 1, ..., m\} - \min\{b_i : i = 0, ..., m\} \ge 0.$ 

Hence,  $x_0 = -\min\{b_i : i = 0, ..., m\}$  and x = 0 is feasible for Q.

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Hence,  $x_0 = -\min\{b_i : i = 0, ..., m\}$  and x = 0 is feasible for Q. It is also a BFS for Q.

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The initial dictionary for  $\mathcal{Q}$  is

$$x_{n+i} = b_i + x_0 - \sum_{j=1}^m a_{ij} x_j$$
  
 $z = -x_0.$ 

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$$x_{n+i} = b_i + x_0 - \sum_{j=1}^m a_{ij} x_j$$
  
 $z = -x_0.$ 

Let  $i_0$  be such that

$$b_{i_0} = \min\{b_i : i = 0, 1, \ldots, m\}.$$

If  $i_0 = 0$ , the LP has feasible origin and so the initial dictionary is optimal.

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If  $i_0 > 0$ , then pivot on this row bringing  $x_0$  into the basis yielding

$$z = b_{i_0} - x_{n+i_0} - \sum_{j=1} a_{i_0j} x_j.$$

If  $i_0 > 0$ , then pivot on this row bringing  $x_0$  into the basis yielding

$$egin{array}{rcl} x_0&=&-b_{i_0}+x_{n+i_0}+\sum\limits_{j=1}^ma_{i_0j}x_j\ x_{n+i}&=&b_i-b_{i_0}+x_{n+i_0}-\sum\limits_{j=1}^m(a_{ij}-a_{i_0j})x_j,\quad i
eq i_0\ z&=&b_{i_0}-x_{n+i_0}-\sum\limits_{j=1}^ma_{i_0j}x_j. \end{array}$$

This dictionary is feasible for  $\mathcal{Q}$ .

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### Initializing the Auxiliary Problem: Example

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## Example

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#### Example

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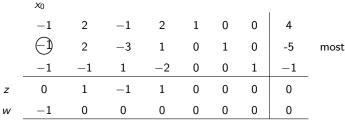
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### First Pivot

	<i>x</i> <sub>0</sub>							
	$^{-1}$	2	-1	2	1	0	0	4
	-1	2 2 —1	-3	1	0	1	0	-5
	-1	-1	1	-2	0	0	1	-1
z	0	1	-1	1	0	0	0	0
w	-1	0	0	0	0	0	0	

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	<i>x</i> <sub>0</sub>							
	-1	2	-1	2	1	0	0	4
	$\bigcirc$	2	-3	1	0	1	0	-5
	-1	-1	1	-2	0	0	1	-1
Ζ	0	1	-1	1	0	0	0	0
w	-1	0	0	0	0	0	0	0
	0	0	2	1	1	-1	0	9
	1	-2	3	-1	0	-1	0	5
	0	-3	4	-3	0	-1	1	4
z	0	1	-1	1	0	0	0	0
w	0	-2	3	-1	0	-1	0	5

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### First Pivot

	<i>x</i> <sub>0</sub>							I
	-1	2	-1	2	1	0	0	4
	-1	2	-3	1	0	1	0	-5
	-1	-1	1	-2	0	0	1	-1
Ζ	0	1	-1	1	0	0	0	0
w	-1	0	0	0	0	0	0	0
	0	0	2	1	1	$^{-1}$	0	9
	1	-2	3	-1	0	-1	0	5
	0	-3	4	-3	0	-1	1	4
z	0	1	-1	1	0	0	0	0
w	0	-2	3	-1	0	-1	0	5

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# Second Pivot

	0	0	2 3 (4)	1	1	$^{-1}$	0	9
	1	-2	3	-1	0	-1	0	5
	0	-3	4	-3	0	-1	1	4
z	0	1	-1 3	1	0	0	0	0
w	0	-2	3	-1	0	-1	0	5

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# Second Pivot

1						
1 -1 0 9	1	1	2	0	0	
0 -1 0 5	0	-1	3	-2	1	
0 -1 1 4	0	-3	4	-3	0	
0 0 0	0	1	-1	1	0	z
0 -1 0 5	0	$^{-1}$	3	-2	0	W
$1  -\frac{1}{2}  -\frac{1}{2}  7$	1	<u>5</u> 2	0	$\frac{3}{2}$	0	
$0  -\frac{1}{4}  -\frac{3}{4}  2$	0	$\frac{5}{4}$	0	$\frac{1}{4}$	1	
$0 -\frac{1}{4} -\frac{1}{4} = 1$	0	$-\frac{3}{4}$	1	$-\frac{3}{4}$	0	
$0 -\frac{1}{4} -\frac{1}{4} = 1$	0	$\frac{1}{4}$	0	$\frac{1}{4}$	0	z
$0  -\frac{1}{4}  -\frac{3}{4}  2$	0	<u>5</u> 4	0	$\frac{1}{4}$	0	w
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0 1 0 0	$\begin{array}{c} -1 \\ \frac{5}{2} \\ \frac{5}{4} \\ -\frac{3}{4} \\ \frac{1}{4} \end{array}$	3 0 0 1 0	$-2$ $\frac{\frac{3}{2}}{\frac{1}{4}}$ $-\frac{3}{4}$	0 0 1 0 0	W z

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# Second Pivot

	0	0	2	1	1	$^{-1}$	0	9
	1	-2	3	-1	0	-1	0	5
	0	-3	4	-3	0	-1	1	4
z	0	1	-1	1	0	0	0	0
w	0	-2	3	-1	0	-1	0	5
	0	$\frac{3}{2}$	0	<u>5</u> 2	1	$-\frac{1}{2}$	$-\frac{1}{2}$	7
	1	$\frac{1}{4}$	0	$\begin{pmatrix} 5\\ \overline{4} \end{pmatrix}$	0	$-\frac{1}{4}$	$-\frac{3}{4}$	2
	0	$-\frac{3}{4}$	1	$-\frac{3}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	1
z	0	$\frac{1}{4}$	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	1
w	0	1	0	5	0	1	3	2

# Third Pivot

	0	$\frac{3}{2}$	0	<u>5</u> 2	1	$-\frac{1}{2}$	$-\frac{1}{2}$	7
	1	$\frac{1}{4}$	0	$\begin{pmatrix} 5\\ \overline{4} \end{pmatrix}$	0	$-\frac{1}{4}$	$-\frac{3}{4}$	2
	0	$-\frac{3}{4}$	1	$-\frac{3}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	1
z	0	$\frac{1}{4}$	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	1
W	0	$\frac{1}{4}$	0	$\frac{5}{4}$	0	$-\frac{1}{4}$	$-\frac{3}{4}$	2
	-2	1	0	0	1	0	1	3
	$\frac{4}{5}$	$\frac{1}{5}$	0	1	0	$-\frac{1}{5}$	$-\frac{3}{5}$	$\frac{8}{5}$
	<u>3</u> 5	$-\frac{3}{5}$	1	0	0	$-\frac{2}{5}$	$-\frac{1}{5}$	<u>11</u> 5
z	$-\frac{1}{5}$	$\frac{4}{20}$	0	0	0	$-\frac{4}{20}$	$\frac{8}{20}$	$\frac{3}{5}$
w	-1	0	0	0	0	0	0	0

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	0 1 0	$\frac{\frac{3}{2}}{\frac{1}{4}}$ $-\frac{3}{4}$	0 0 1	$ \begin{array}{r} \frac{5}{2} \\ \frac{5}{4} \\ -\frac{3}{4} \end{array} $	1 0 0	$-\frac{1}{2}$ $-\frac{1}{4}$ $-\frac{1}{4}$	$-\frac{1}{2}$ $-\frac{3}{4}$ $\frac{1}{4}$	7 2 1	
Ζ	0	$\frac{1}{4}$	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	1	
W	0	$\frac{1}{4}$	0	$\frac{5}{4}$	0	$-\frac{1}{4}$	$-\frac{3}{4}$	2	
	-2	1	0	0	1	0	1	3	Auxiliary
	$\frac{4}{5}$	$\frac{1}{5}$	0	1	0	$-\frac{1}{5}$	$-\frac{3}{5}$	8 5	problem
	<u>3</u> 5	$-\frac{3}{5}$	1	0	0	$-\frac{2}{5}$	$-\frac{1}{5}$	<u>11</u> 5	solved.
Ζ	$-\frac{1}{5}$	$\frac{4}{20}$	0	0	0	$-\frac{4}{20}$	$\frac{8}{20}$	3 5	
W	-1	0	0	0	0	0	0	0	
z	$-2$ $\frac{4}{5}$ $\frac{3}{5}$ 1	$\begin{array}{c}1\\\frac{1}{5}\\-\frac{3}{5}\\\frac{4}{20}\end{array}$	0 0 1 0	0 1 0 0	1 0 0 0	$-\frac{1}{5}$ $-\frac{2}{5}$ $-\frac{4}{20}$	$ \begin{array}{c} 1 \\ -\frac{3}{5} \\ -\frac{1}{5} \\ \frac{8}{20} \end{array} $	3 <sup>8</sup> 5 <u>11</u> <u>3</u> 5 3	problem

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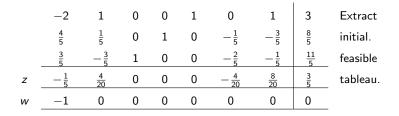
									-
	-2	1	0	0	1	0	1	3	
	$\frac{4}{5}$	$\frac{1}{5}$	0	1	0	$-\frac{1}{5}$	$-\frac{3}{5}$	<u>8</u> 5	
	$\frac{3}{5}$	$-\frac{3}{5}$	1	0	0	$-\frac{2}{5}$	$-\frac{1}{5}$	$\frac{11}{5}$	
z	$-\frac{1}{5}$	$\frac{4}{20}$	0	0	0	$-\frac{4}{20}$	$\frac{8}{20}$	<u>3</u> 5	Original LP
w	-1	0	0	0	0	0	0	0	is feasible.

Lecture 8: Initialization and the Two Phase Simplex A

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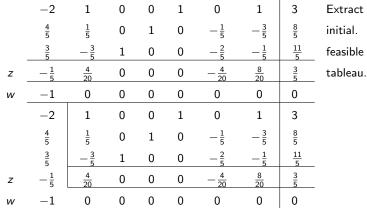
#### Extract Initial Feasible Tableau



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# Third Pivot



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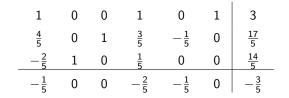
	1	0	0	1	0	Ð	3
	$\frac{1}{5}$	0	1	0	$-\frac{1}{5}$	$-\frac{3}{5}$	85
	$-\frac{3}{5}$	1	0	0	$-\frac{2}{5}$	$-\frac{1}{5}$	$\frac{11}{5}$
	1 5	0	0	0	$-\frac{1}{5}$	25	35
-	1	0	0	1	0	1	3
	$\frac{4}{5}$	0	1	$\frac{3}{5}$	$-\frac{1}{5}$	0	$\frac{17}{5}$
	$-\frac{2}{5}$	1	0	$\frac{1}{5}$	0	0	$\frac{\frac{17}{5}}{\frac{14}{5}}$
	$-\frac{1}{5}$	0	0	$-\frac{2}{5}$	$-\frac{1}{5}$	0	$-\frac{3}{5}$

Lecture 8: Initialization and the Two Phase Simplex A

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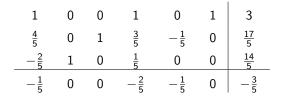
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#### Phase II: Solution



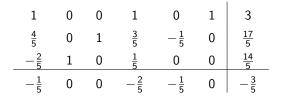
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$$\left(\begin{array}{c} x_1\\ x_2\\ x_3 \end{array}\right) = \left(\begin{array}{c} 0\\ 2.8\\ 3.4 \end{array}\right)$$

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$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 2.8 \\ 3.4 \end{pmatrix} \qquad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.2 \\ 0 \end{pmatrix}$$

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$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 2.8 \\ 3.4 \end{pmatrix} \qquad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.2 \\ 0 \end{pmatrix}$$

with optimal value z = .6.

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# Steps for Phase I of the Two Phase Simplex Algorithm

We assume 
$$b_{i_0} = \min\{b_i : i = 1, \dots, m\} < 0.$$
  
Form the standard initial tableau:  $\begin{bmatrix} 0 & A & I & | & b \\ \hline -1 & c & 0 & | & 0 \end{bmatrix}$ .  
Border the initial tableau:  $\begin{bmatrix} -1 & 0 & A & I & | & b \\ \hline 0 & -1 & c & 0 & | & 0 \\ \hline -1 & 0 & 0 & 0 & | & 0 \end{bmatrix}$ .

- In the first pivot, the pivot row is the i<sub>0</sub> row and the pivot column is the first column (the x<sub>0</sub> column).
- Apply simplex algorithm on the *w* row until optimality.
- If optimal value is positive, stop the original LP is not feasible.
- If the optimal value is zero, extract feasible tableau for the original problem and pivot to optimality.

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Use the two phase simplex method to solve the following LP:

Hint: A complete solution is possible in 3 pivots.

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