FINAL EXAM OUTLINE FOR MATH 407

EXAM DATES:

- Sections AC: Wednesday, December 16, 2020: 2:30 - 4:20pm.
- Sections BD: Thursday, December 17, 2020: 8:30 - 10:20am.

EXAM OUTLINE

The final exam will consist of 5 questions. The first 4 questions are each worth 65 points and the last question is worth 90 points for a total of 350 points. Questions 1 and 2 will be given during class time on Friday, December 11 and the remaining three questions will be given on the assigned final exam date. The content of each question is as follows.

Question 1: (Two Phase Simplex Algorithm: 65 Points) In this question you will be given an LPs and asked to solve it using the Two Phase Simplex Algorithm.

You will need to show all of your work to get full credit. In addition, you may be asked to answer a question about the nature of the solution that you have found or the nature of the dual solution, e.g. describe the entire optimal solution set to the primal (dual) problem when more than one optimal solution exists.

Question 2: (Dual Simplex Algorithm: 65 Points) In this question you will be given an LPs and asked to solve it using the Dual Simplex Algorithm.

You will need to show all of your work to get full credit. In addition, you may be asked to answer a question about the nature of the solution that you have found or the nature of the dual solution, e.g. describe the entire optimal solution set to the primal (dual) problem when more than one optimal solution exists.

Question 3: (General Duality: 65 Points) In this question you will be asked to formulate the dual of one or more LPs without first bringing it to standard form.

Question 4: (Geometric Duality: 65 Points) In this question you will be asked to apply the Geometric Duality Theorem to determine if a given vector solves a given LP.

Question 5: (LP Sensitivity Analysis: 90 Points) In this problem you will be given an LP model, its initial tableau, and an associated optimal tableau. You will then be asked to answer a list sensitivity analysis questions about the problem and it's optimal solution using the techniques developed in the course including the computation of break-even prices, range analysis, and pricing out. Answering these questions properly may require some combination of primal and/or dual simplex pivoting.

SAMPLE QUESTIONS

1. Use the Two Phase Simplex Algorithm to solve the following LPs stating their solution, the solution to their duals, and their optimal values. (Solution methods other than the Two Phase Simplex Algorithm will be given zero credit)

   (a)

$$\begin{array}{lrcrcrcr} \text{maximize} & x_1 & + & x_2 & + & 3x_3 & & \\ \text{subject to} & x_1 & - & x_2 & - & 2x_3 & \leq & -2 \\ & x_1 & + & 2x_2 & + & 2x_3 & \leq & 2 \\ & 0 & \leq & x_1, & x_2, & x_3. & & \end{array}$$

   (b)

$$\begin{array}{lrcrcr} \text{maximize} & x_1 & + & 4x_2 & & \\ \text{subject to} & x_1 & + & x_2 & \leq & 1 \\ & x_1 & - & x_2 & \leq & -2 \\ & 0 & \leq & x_1, & x_2. & \end{array}$$

2. Use the Dual Simplex Algorithm to solve the following LP stating the solution, the solution to the duals, and the optimal value. (Solution methods other than the Dual Simplex Algorithm will be given zero credit)

$$\begin{array}{lrcrcrcrcr} \text{maximize} & -2x_1 & - & 2x_2 & - & x_3 & - & 5x_4 & & \\ \text{subject to} & 2x_1 & - & x_2 & + & x_3 & - & x_4 & \leq & 4 \\ & & & x_2 & + & 2x_3 & - & x_4 & \leq & 5 \\ & x_1 & - & x_2 & - & x_3 & - & x_4 & \leq & -3 \\ & 0 & \leq & x_1, & x_2, & x_3, & x_4. & & & \end{array}$$

3. Formulate a dual for the following LPs.

   (a)

$$\begin{array}{lrcl} \text{minimize} & c^T x & & \\ \text{subject to} & Ax & \leq & 0 \\ & Bx & = & 0, \end{array}$$

   where $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{s\times n}$, and $B \in \mathbb{R}^{t\times n}$.

   (b)

$$\begin{array}{ll} \text{maximize} & 2x_1 - 3x_2 + 10x_3 \\ \text{subject to} & x_1 + x_2 - x_3 = 12 \\ & x_1 - x_2 + x_3 \leq 8 \\ & 0 \leq x_2 \leq 10 \end{array}$$

4. Use the Geometric Duality Theorem to determine if the vector $x = (0, 5, 0, 1, 1)^T$ solves the LP

$$\begin{array}{lrcrcrcrcrcr} \text{maximize} & & & x_2 & & & + & 5x_4 & + & 5x_5 & & \\ \text{subject to} & x_1 & + & 2x_2 & - & x_3 & + & x_4 & & & \leq & 11 \\ & 3x_1 & + & x_2 & + & 4x_3 & + & x_4 & + & x_5 & \leq & 10 \\ & 2x_1 & - & x_2 & + & 2x_3 & + & x_4 & + & 2x_5 & \leq & -2 \\ & x_1 & & & & & + & x_4 & + & 3x_5 & \leq & 4 \\ & 0 & \leq & x_1, & & x_2, & & x_3, & & x_4, & & x_5 \end{array}$$

5. (a) (Silicon Chip Corp) A Silicon Valley firm specializes in making four types of silicon chips for personal computers. Each chip must go through four stages of processing before completion. First

the basic silicon wafers are manufactured, second the wafers are laser etched with a micro circuit, next the circuit is laminated onto the chip, and finally the chip is tested and packaged for shipping. The production manager desires to maximize profits during the next month. During the next 30 days she has enough raw material to produce 4000 silicon wafers. Moreover, she has 600 hours of etching time, 900 hours of lamination time, and 700 hours of testing time. Taking into account depreciated capital investment, maintenance costs, and the cost of labor, each raw silicon wafer is worth \$1, each hour of etching time costs \$40, each hour of lamination time costs \$60, and each hour of inspection time costs \$10. The production manager has formulated her problem as a linear program with the following initial tableau:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b
raw wafers	100	100	100	100	1	0	0	0	4000
etching	10	10	20	20	0	1	0	0	600
lamination	20	20	30	20	0	0	1	0	900
testing	20	10	30	30	0	0	0	1	700
	2000	3000	5000	4000	0	0	0	0	0

where x_1, x_2, x_3, x_4 represent the number of 100 chip batches of the four types of chips. After solving by the Simplex Algorithm, the final tableau is:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b
0.5	1	0	0	.015	0	0	$-.05$	25
-5	0	0	0	$-.05$	1	0	$-.5$	50
0	0	1	0	$-.02$	0	.1	0	10
0.5	0	0	1	.015	0	$-.1$	.05	5
-1500	0	0	0	-5	0	-100	-50	$-145,000$

Answer each of the following questions as if it were a separate event. Do not consider the cumulative effects between problems.

i. A new product is to be considered for production. This chip requires 25 hours of etching, 20 hours lamination, and 20 hours testing time per 100 chip batch. What is the breakeven sale price for this chip, that is, at what sale price does it become efficient to introduce this product into the optimal production schedule?

ii. A competitor has just come out with a chip that serves the same market as our type 4 chip. A price war is imminent. By how much can we reduce the profitability of this chip and yet have it remain in the optimal production schedule?

iii. A flu is going around and has hit the shop pretty bad, causing an across the board 10% reduction in the etching, lamination, and testing time. What effect will this have on the optimal production schedule and profitability?

(b) Concrete Products Corporation has the capability of producing four types of concrete blocks. Each block must be subjected to four processes: batch mixing, mold vibrating, inspection, and yard drying. The plant manager desires to maximize profits during the next month. During the upcoming 30 days, he has 800 machine hours available on the batch mixer, 1000 hours on the mold vibrator, and 340 man-hours of inspection time. Yard-drying time is unconstrained. Taking into consideration depreciated capital investment and maintenance costs, batch mixing time is worth \$5 per hour, mold vibrating time is worth \$10 per hour, and inspection time is worth \$10 per hour, and the materials costs for the blocks are \$50, \$80, \$100, and \$120 per pallet, respectively. The production director has formulated his problem as a linear program with the following initial tableau:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
batch mixing	1	2	10	16	1	0	0	800
mold vibrating	1.5	2	4	5	0	1	0	1000
inspection	0.5	0.6	1	2	0	0	1	340
	80	140	300	500	0	0	0	0

where x_1, x_2, x_3, x_4 represent the number of pallets of the four types of blocks. The cost coefficients in the z-row represent the profit in dollars per pallet (not revenue!!). After solving by the Simplex method, the final tableau is:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

QUESTIONS FOR THE CONCRETE PRODUCTION PROBLEM

Answer each of the following questions as if it were a separate event. Do not consider the cumulative effects between problems.

i. How much must a pallet of type 3 blocks be sold for in order to make it efficient to produce them?

ii. What is the minimum price at which type 2 blocks can be sold and and yet maintain them in the optimal production mix?

iii. If the 800 machine hours on the batch mixer is uncertain, for what range of hours of batch mixing time is it efficient for the optimal production mix to consist of type 1 and 2 blocks?

iv. A competitor has offered the manager additional batch mixing time at $30 an hour. Neglecting transportation costs, should the manager accept this offer and if so, how many hours of batch mixing time should he purchase at this price?

v. The market for type 2 blocks has gotten hot lately. We can now sell them for $30 more than we used to. If we make this increase, what is the new optimal production schedule ?

vi. The mold vibrator needs major repairs. Consequently, we will lose 300 hours of mold vibrating time this month. What should be the new production schedule for this month ?

vii. We intend to introduce a new type of block. This block requires 4 hours of batch mixing time, 4 hours mold vibrating time, and 1 hour of inspection time per pallet. The materials costs for this type of block are $80 per pallet. At what price must this product be sold in order to make it efficient to produce ?