REVIEW QUESTIONS FOR FIRST EXAM

1) Let $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & 1 & 5 \\ 1 & 2 & 6 \end{bmatrix}$.. Let $V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $V_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, $V_3 = \begin{bmatrix} 5 \\ 5 \\ 6 \end{bmatrix}$.

(a) Find A^{-1} if it exists, or explain why A^{-1} doesn't exist.

(b) Find scalars $c_1, c_2,$ and c_3 such that $c_1V_1 + c_2V_2 + c_3V_3 = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$.

(c) A certain matrix B has the property that:

$$BV_1 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix},\ BV_2 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix},\ BV_3 = \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix}.$$

Based on this information, compute $B\begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$.

(d) Based on the information in part (c), can you determine if B is singular or non-singular? Explain your answer briefly.

(e) Compute the product BA.

(f) Find the matrix B used in this problem.

2) Consider the linear system $Ax = b$.

(a) Suppose that A is a 3×5 matrix and that b is a vector in $\boldsymbol{R}^3$. Without knowing anything more about the matrix A or the vector b, what can you say about the number of solutions to the matrix equation $Ax = b$?

(b) Now suppose that A is a 5×3 matrix and $A\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

Without knowing anything else about A, what can you say about the rank of A? Explain your answer briefly.

3) Suppose that A and B are invertible 3×3 matrices.

(a) Find the simplest expression possible for $(BA)(BABA^{-1}B^{-1})^{-1}(BA)$.

(b) Suppose that you are now told that A and B are row-equivalent to each other. Based on that information, can you now simplify the expression you found in part (a)? Explain.

(c) Suppose you are told that $A\begin{bmatrix}3\\7\\2\end{bmatrix}=\begin{bmatrix}3\\7\\2\end{bmatrix}$. Based on that information, what can you say (if anything) about the matrix $A-I_3$? Explain.

4) Let $A=\begin{bmatrix}1&3&0\\1&4&3\\1&3&1\end{bmatrix}$.

(a) Are the columns of A linearly independent or linearly dependent? Show your work.

(b) Find a matrix B such that $BA=\begin{bmatrix}7&7&2\\9&9&5\end{bmatrix}$.

5) (a) Solve the system: $\begin{aligned}x_1+x_2+x_4&=1\\2x_1+2x_2+3x_4&=1\end{aligned}$ (Give your answer in vector form).

(b) Let A be the coefficient matrix of the system in part (a), and let $b=\begin{bmatrix}1\\1\end{bmatrix}$. The system $Ax=b$ has infinitely many solutions. Suppose that $x=u_1$ and $x=u_2$ are two different solutions. Let $v=u_1-u_2$. What is Av?

6) (a) Find a parabola of the form $y=ax^2+bx+c$ passing through the points (0,-1), (1,3), and (2,6).

(b) Find all values of a for which the following homogenous system has non-trivial solutions.

$$\begin{aligned}(1-a)x\ +\ z&=0\\-ay+z&=0\\y-az&=0\end{aligned}$$