ERRATA TO "REAL ANALYSIS," 2nd edition

(6th and later printings)

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Additional corrections will be gratefully received at `folland@math.washington.edu` .

Page 7, line 12: $Y \cup \{y_0\} \quad\to\quad B \cup \{y_0\}$

Page 7, line -12: $X \in \quad\to\quad x \in$

Page 8, next-to-last line of proof of Proposition 0.10: $E \quad\to\quad X$

Page 12, line 17: $a \in \mathbb{R} \quad\to\quad x \in \mathbb{R}$ (two places)

Page 14, line 16: $x \in X \quad\to\quad x \in X_1$

Page 14, line 17: whenver $\to$ whenever

Page 22, line 2: susbset $\to$ subset

Page 24, Exercise 1, line 1: A family $\to$ A nonempty family

Page 24, Exercise 3a: disjoint $\to$ disjoint nonempty

Page 29, Proposition 1.10: The hypothesis that $X \in \mathcal{E}$ was included only to guarantee that $\mu^*(A)$ is well-defined for all $A \subset X$, and with the understanding that $\inf(\varnothing) = +\infty$, it is unnecessary. The proof extends to the general case without change, as the condition $\mu^*(\bigcup A_j) \le \sum \mu^*(A_j)$ is nontrivial only when $\mu^*(A_j) < \infty$ for all j.

Page 34, line 1: $\bigcup_1^n J_j \quad\to\quad \bigcup_1^m J_j$

Page 35, line -3: open h-intervals $\to$ open intervals

Page 37, line -1: countable $\to$ countable set.

Page 38, line -4: $\sum_0^\infty \quad\to\quad \sum_1^\infty$

Page 40, line 2 of §1.6: 2.7 $\to$ 2.8

Page 45, line 5: $[\infty, \infty] \quad\to\quad [-\infty, \infty]$

Page 45, line 8: 2.3 $\to$ 1.2

Page 47, Figure 2.1: The graph of ϕ_1 should have an extra "step" where the ordinate goes from 1 to $\frac{3}{2}$ and then from $\frac{3}{2}$ to 2, rather than directly from 1 to 2.

Page 49, line -8: inegrals $\to$ integrals

Page 56, last line of proof of Theorem 2.27: $(x, t) \quad\to\quad (x, t_0)$

Page 60, Exercise 27c: $\log(b/a) \quad\to\quad \log(a/b)$

Page 60, Exercise 31e: $s^2 \quad\to\quad a^2$

Page 61, line 9: repectively $\to$ respectively

Page 66, line -4: $\bigcap_1^\infty E_n \quad\to\quad E = \bigcap_1^\infty E_n$

Page 67, next-to-last line of Theorem 2.37: $\int f^y \, d\nu \quad\to\quad \int f^y \, d\mu$.

Page 69, Exercise 49a: $\mathcal{M} \times \mathcal{N} \quad \to \quad \mathcal{M} \otimes \mathcal{N}$

Page 69, Exercise 50: Either assume $f < \infty$ everywhere or use the condition $y < f(x)$ to define G_f. Also, $\mathcal{M} \times \mathcal{B}_{\mathbb{R}} \quad \to \quad \mathcal{M} \otimes \mathcal{B}_{\mathbb{R}}$.

Page 70, proof of Theorem 2.40, line 2: rectangles $\to$ rectangles, which may be assumed bounded,

Page 72, line 5: definitons $\to$ definitions

Page 75, line 9: $\sum_j (x_j - a_j)(\partial g/\partial x_j)(y) \quad \to \quad \sum_k (x_k - a_k)(\partial g_j/\partial x_k)(y)$

Page 75, line 9: joning $\to$ joining

Page 76, line 6: $\bigcup_1^\infty U_j \quad \to \quad \bigcap_1^\infty U_j$

Page 76, line -7: $f \circ G \quad \to \quad f \circ G|\det DG|$

Page 76, line -5: $G(\Omega)) \quad \to \quad G(\Omega)$

Page 77, Exercise 58: $\int \quad \to \quad \int_0^\infty$

Page 79, line 2: $(a, b] \times E \quad \to \quad (a, b] \times E$ or $(a, \infty) \times E$

Page 87, line 3: $\nu(A_j) > \sum \quad \to \quad \nu(A_j) \geq \sum$

Page 88, Exercise 3c: $|f| \leq 1 \quad \to \quad |f| \leq 1$ and $\int_E f\, d\nu$ exists [The latter condition is automatic when ν is finite.]

Page 88, Exercise 6: $\int f\, d\mu \quad \to \quad \int_E f\, d\mu$

Page 90, line -6: $f \quad \to \quad f_j$

Page 102: (3.24) should be interpreted as "$T_F(b) = T_F(a) + \sup\{\dots\}$" in the case $T_F(b) = T_F(a) = \infty$.

Page 103, line -5: $\pm\frac{1}{2}F(-\infty) \quad \to \quad \mp\frac{1}{2}F(-\infty)$

Page 104, line 7 of proof of Lemma 3.28: $x_0 < \cdots \quad \to \quad x = x_0 < \cdots$

Page 104, line -12: $\sum_1^n \quad \to \quad \sum_1^m$

Page 105, line 2 of proof of Proposition 3.32: suppose that E $\to$ suppose that F is absolutely continuous and E

Page 105, line 5 of proof of Proposition 3.32: $\mu(U_j) < \delta \quad \to \quad m(U_j) < \delta$

Page 106, line 4: greatest integer less than $\delta^{-1}(b - a) + 1$ $\to$ smallest integer greater than $\delta^{-1}(b - a)$

Page 107, Exercise 28b: $\mu_{T_F(E)} \quad \to \quad \mu_{T_F}(E)$

Page 115, line -12: Propostiion $\to$ Proposition

Page 120, line -2: a neighborhood $\to$ an open neighborhood

Page 125, line 16: is a set $\to$ is a nonempty set

Page 144, line 12: an LCH $\to$ a noncompact LCH

Page 145, paragraph after the end-of-proof sign, line 3: locally compact $\to$ locally compact and noncompact

Page 146, Exercise 73: In the definition of completely regular algebra, add the condition that the algebra be closed under complex conjugation. Also, in parts (a), (b), and (d), the word

“Hausdorff” is redundant since it is incorporated in the definition of “compactification” on p. 144.

Page 146, Exercise 73c: contains $\mathcal{F}$ $\rightarrow$ contains $\mathcal{F}$ and the constant functions

Page 146, Exercise 73d: Insert “(up to homeomorphisms)” after “of X”.

Page 159, next-to-last line of proof of Theorem 5.8: Moroever $\rightarrow$ Moreover

Page 165, line 6: $x \in X$ $\rightarrow$ $x \in \mathcal{X}$

Page 166, line -2 of proof of Theorem 5.14: $(1-t)x + (1-t)z$ $\rightarrow$ $(1-t)x - (1-t)z$

Page 166, line -1: $U_{x\alpha_j\epsilon_j}$ $\rightarrow$ $U_{0\alpha_j\epsilon_j}$

Page 167, line 3: $p_{\alpha_j}(y) < \epsilon$ $\rightarrow$ $p_{\alpha_j}(y) \leq \epsilon$

Page 167, bulleted item at bottom (continuing to next page): $\mathbb{C}^X$ should be replaced by the space of locally bounded functions on X, i.e., the space of all complex-valued functions f on X such that $p_K(f) < \infty$ for all K.

Page 174, line 2: paralellogram $\rightarrow$ parallelogram

Page 174, lines -8 and -4: $\mathcal{X}$ $\rightarrow$ $\mathcal{H}$

Page 177, line 1: e_α $\rightarrow$ u_α and $\mathcal{X}$ $\rightarrow$ $\mathcal{H}$

Page 179, next-to-last line of notes for §5.1: coincides with $\rightarrow$ extends

Page 179, line -2: $x_1 \in \mathcal{X}_0$ $\rightarrow$ $x_0 \in \mathcal{X}_0$

Page 194, line -3, “simple consequence”: Actually, all the y-sections of the set $\{(x,y) : |f(x,y)| > \|f(\cdot,y)\|_\infty\}$ have μ-measure 0, and you need Tonelli to deduce that μ-almost all the x-sections have ν-measure 0.

Page 196, Exercise 28b: $J_1 f$ $\rightarrow$ $J_\alpha f$ (Hint: Focus on the behavior of $J_\alpha f(x)$ as $x \rightarrow \infty$.)

Page 197, line -2: on $(0,\infty)$, $\rightarrow$ on $[0,\infty)$ such that $\phi(0) = 0$,

Page 203, statement of Marcinkiewicz interpolation theorem, last sentence: If $p_0 = p_1$ (so p doesn’t vary), B_p and $|p - p_j|$ should be replaced by B_q and $|q - q_j|$.

Page 204, last line of (6.33): $C_j^{p_j}$ $\rightarrow$ $C_j^{q_j}$

Page 206, Theorem 6.36, line 4: $1 \leq p < \infty$ $\rightarrow$ $1 \leq p < q/(q-1)$

Page 208, Exercise 41: For the case $p = \infty$, assume μ semifinite.

Page 208, Exercise 45, lines 3 and 4: T is weak type $(1, n\alpha^{-1})$ and strong type (p, r) where $1 < p < n(n-\alpha)^{-1}$ and $r^{-1} = p^{-1} - (n-\alpha)n^{-1}$.

Page 210, final sentence: Theorem 6.36 was discovered independently, a little earlier than [51], by D. R. Adams (A trace inequality for generalized potentials, *Studia Math.* **48** (1973), 99–105).

Page 212, line 13: a Borel measure $\rightarrow$ a Borel measure that is finite on compact sets

Page 217, lines 7 and 8: f $\rightarrow$ f_1

Page 218, line -5: χ_u $\rightarrow$ χ_U

Page 221, Exercise 15e: $E \subset \mathcal{B}_{\Omega^*}$ $\rightarrow$ $E \in \mathcal{B}_{\Omega^*}$

Page 224, line 8: Insert minus signs before the two middle integrals.

Page 224, line 9: $\int f_n\,d\mu \quad\to\quad \int f\,d\mu_n$

Page 224, line -4 of proof of Proposition 7.19: $(-\infty, N] \quad\to\quad (-\infty, -N]$

Page 224, Exercise 18, line 1: $\mathcal{M}(X) \quad\to\quad M(X)$

Page 225, Exercise 24b: $\int f\,d\mu \quad\to\quad 0$

Page 225, Exercise 24c: $F(x) \quad\to\quad 0$

Page 225, Exercise 27: k functionals $\quad\to\quad$ k bounded functionals

Page 226, proof of Theorem 7.20, next-to-last line: $\pi_1(K)\times\pi_2(K) \quad\to\quad \pi_X(K)\times\pi_Y(K)$

Page 226, proof of Theorem 7.20, last line: $= \quad\to\quad \leq$

Page 226, line 2 of Proposition 7.21: $X\otimes Y \quad\to\quad X\times Y$

Page 226, line -2: $\overline{U}\times\overline{V} \quad\to\quad U\times V$

Page 227, 4th and 3rd lines before Lemma 7.23: Replace the clause "Exercises 12 and . . . $\mu\widehat{\times}\nu$" by "Exercise 12 shows that $\mu\widehat{\times}\nu(\{0\}\times\mathbb{R}) \neq 0 = \mu\times\nu(\{0\}\times\mathbb{R})$". (The semifinite part of $\mu\widehat{\times}\nu$ disagrees with $\mu\times\nu$ on $\{(x,x) : x\in[0,1]\}$; see Exercise 2.46.)

Page 228, line 3: $\bigcap_1^m \quad\to\quad \bigcap_1^n$

Page 229, line -10: $\mathcal{B}_X\times\mathcal{B}_Y \quad\to\quad \mathcal{B}_X\otimes\mathcal{B}_Y$

Page 232, line 5 of paragraph 3: $L^1(\overline{\mu}) \quad\to\quad L^1(\overline{\mu})^*$

Page 242, line 12: $\|g\|_{(N+n+1,\alpha)} \quad\to\quad \|g\|_{(N+n+1,0)}$

Page 246, Exercise 9: Assume $p<\infty$.

Page 247, line 2 of Theorem 8.19: $\mathbb{T}^n \quad\to\quad \mathbb{Z}^n$

Page 250, line -2: $\sum_{|\gamma|\leq|\beta|}\|f\|_{(N+n+1,\gamma)} \quad\to\quad \sum_{|\gamma|\leq N}\|f\|_{(|\beta|+n+1,\gamma)}$

Page 251, line 4: $-2\pi a e^{-\pi a x^2} \quad\to\quad -2\pi a x e^{-\pi a x^2}$

Page 254, line 5: $\mathbb{Z}^N \quad\to\quad \mathbb{Z}^n$

Page 254, line 4 of proof of Theorem 8.32: 8.35 $\quad\to\quad$ 8.31

Page 255, Exercise 16a: $\|f\|_u \quad\to\quad \|f_k\|_u$

Page 256, line 1: right $\quad\to\quad$ left

Page 257, first paragraph of §8.4: To conclude that $f = g$, one needs the injectivity of the Fourier transform on $L^1(\mathbb{T}^n)$. There are several ways to establish this without invoking Fourier inversion, e.g.: (1) Show that if $\widehat{f}=0$ then f defines the zero functional on $C(\mathbb{T}^n)$. (2) Use Exercise 28 (p. 262); from part (a), if $\widehat{f}=0$ then $A_r f = 0$, and from part (b), the mass of P_r concentrates at the origin as $r\to 1$, whence $\|A_r f - f\|_1 \to 0$.

Page 259, line 9: $f_2 * \phi_t(\xi) \quad\to\quad f_2*\phi_t(x)$

Page 259, line 3 of proof of Theorem 8.36: The sum on the right should be $\sum_{\kappa\in\mathbb{Z}^n}$.

Page 261, line 7: $e^{-2\pi i\kappa x} \quad\to\quad e^{2\pi i\kappa x}$

Page 264, line 4: $e^{2\pi(2m+1)x} \quad\to\quad e^{2\pi i(2m+1)x}$

Page 268, formula (8.46): $\frac{1}{2}-x-[x] \quad\to\quad \frac{1}{2}-x+[x]$

Page 269, line 6: $S_m(a_j) \quad \to \quad S_m f(a_j)$

Page 269, Exercise 35a: $\phi \quad \to \quad \phi_m$ (two places)

Page 272, Exercise 39: On line 2, positive $\quad \to \quad$ nonnegative. Also, replace line 3 by the following: at $\frac{\alpha}{m}, \frac{\alpha+1}{m}, \frac{\alpha+m-1}{m}$ for some $\alpha \in [0, 1)$ and $m \in \mathbb{N}$, in which case $\widehat{\mu}(jm) = e^{-2\pi i j\alpha}$ for all $j \in \mathbb{Z}$.

Page 273, line 7: if for all $\quad \to \quad$ for all

Page 274, line -1: $(t^2 + |x|^2)^{-(n+1)/2} \quad \to \quad (t^2 + |x|^2)^{(n+1)/2}$

Page 276, Exercise 43: $e^{-|x|/2} \quad \to \quad \frac{1}{2}e^{-|x|}$

Page 286, line 3: $\phi(y) \quad \to \quad \phi(x)$

Page 286, lines -13 and -5, and page 287, lines 1 and 3: $U \quad \to \quad V$

Page 288, line -10: $\psi(\epsilon x) \quad \to \quad \psi(x/\epsilon)$

Page 289, Exercise 7, line 2: f agrees $\quad \to \quad$ there exists a constant c such that $f + c$ agrees

Page 291, Exercise 13: $f * \psi_t \quad \to \quad F * \psi_t$

Page 293, line -2: $(1 + |x|)^N \quad \to \quad (1 + |x|)^{-N}$

Page 293, line -1: $\|\phi\|_{(0,N)} \quad \to \quad \|\phi\|_{(N,0)}$

Page 294, line 3: by (ii) $\quad \to \quad$ by the preceding example

Page 296, line -9: $x_j \quad \to \quad \xi_j$

Page 297, line 7: One $\quad \to \quad$ On

Page 297, proof of Proposition 9.14, line 3: $f = \widehat{g} \quad \to \quad f = g^\vee$

Page 297, line -3: $\widehat{f}(\kappa) \quad \to \quad \widehat{F}(\kappa)$

Page 300, Exercise 28, line 2: $|\xi|^{\alpha-2} \quad \to \quad |x|^{\alpha-2}$

Page 303, lines 5–6: Fourier transform is $\widehat{g}(\xi) \quad \to \quad$ inverse Fourier transform is $g^\vee(\xi)$

Page 303, line 7: $(1 + |\xi|^2)^s \quad \to \quad (1 + |\xi|^2)^{-s}$

Page 309, Exercise 34c: $\Lambda_a \quad \to \quad \Lambda_\alpha$. Also, apologies for the two conflicting uses of the letter α; one might prefer to replace ∂^α and $|\alpha|$ by ∂^β and $|\beta|$.

Page 320, line -1: the the $\quad \to \quad$ the

Page 323, line 5: $\limsup n^{-1}|S_n| < \epsilon \quad \to \quad \limsup n^{-1}|S_n| \le \epsilon$

Page 325, Exercise 17, line 2: smaple $\quad \to \quad$ sample

Page 325, Exercise 17, line 9: $X_j - M_j \quad \to \quad X_j - M_n$

Page 325, line 3 of §10.3: $e^{(t-\mu)^2/2\sigma^2} \quad \to \quad e^{-(t-\mu)^2/2\sigma^2}$

Page 326, line -6: $X_n \quad \to \quad X_j$

Page 331, line -7: $\exp(\cdots) \quad \to \quad \exp(-\cdots)$

Page 332, formula (10.23): $\exp(\cdots) \quad \to \quad \exp(-\cdots)$

Page 341, proof of Proposition 11.3, line 3: it $\quad \to \quad$ if

Page 344, proof of Theorem 11.9, end of line 2: Delete "$h \in C_c^+$ and".

Page 348, Exercise 9c: In general it is not μ that is decomposable but rather its extension $\overline{\mu}$ to the σ-algebra of μ^*-measurable sets as explained on p. 215.

Page 349, line 3: $\mu^*(A) \cup \mu^*(B) \quad \to \quad \mu^*(A) + \mu^*(B)$

Page 349, line −11: $B^{2k-3} \quad \to \quad B_{2k-3}$

Page 349, line −7: $\sum_{n+1}^{\infty} \quad \to \quad \sum_{n}^{\infty}$

Page 350, proof of Proposition 11.17: Concerning the applicability of Proposition 1.10, see the correction to Page 29 in this errata list.

Page 357, Figure 11.1(b): In the bottom figure, the small triangle in the center should not be shaded.

Page 358, line 10: $C_(X) \quad \to \quad C(X)$

Page 358, line −7: $x_{i_1 \cdots x_k} \quad \to \quad x_{i_1 \cdots i_k}$

Page 362, first display: $\dfrac{\partial y_i}{\partial x_k}\dfrac{\partial y_j}{\partial x_l} \quad \to \quad \dfrac{\partial y_k}{\partial x_i}\dfrac{\partial y_l}{\partial x_j}$

Page 373, reference 131: of $\quad \to \quad$ in

Page 373, reference 139: *in* $\quad \to \quad$ *on*

Page 378, line −2: $CS' \quad \to \quad \mathcal{S}'$