Note on Munkres's Problem 16.5

Munkres's statement of this problem is misleading, because the first sentence ("Let X and X' denote a single set in the topologies $\mathcal{T}$ and $\mathcal{T}'$, respectively") suggests that we're considering X and X' as *elements* of the given topologies. What he really meant to say, instead of "in the topologies," was something more like "endowed with the topologies."

Here is a more carefully worded version of the same problem. Unless you've already answered the problem as stated below, please redo your solution to this problem and hand it in on Friday, October 29.

§16, Problem 5 reworded:

Let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on a given set X_0. Let X denote the topological space consisting of the set X_0 endowed with the topology $\mathcal{T}$, and let X' denote the same set X_0 with the topology $\mathcal{T}'$. Similarly, let Y and Y' denote another set Y_0 endowed with two topologies $\mathcal{U}$ and $\mathcal{U}'$, respectively. Assume these sets are nonempty.

(a) Show that if $\mathcal{T}' \supset \mathcal{T}$ and $\mathcal{U}' \supset \mathcal{U}$, then the product topology on $X' \times Y'$ is finer than the product topology on $X \times Y$.

(b) Does the converse of (a) hold? Justify your answer.