Math 442 **Differential Geometry** **Winter 2011**

SYLLABUS

Lectures: MWF 12:30–1:20
Lowe 113

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Course Web site: *www.math.washington.edu/~lee/Courses/442-2011*
(or from the Math Department home page,
Class Web Pages → Math 442A)

Required Textbook:

- [B] Christian Bär, *Elementary Differential Geometry*, Cambridge University Press, 2010.

Reserve Books:

For further information, you are encouraged to consult the following sources (all on reserve in the Mathematics Research Library):

- D. Lay, *Linear Algebra and Its Applications*, Addison-Wesley, 2000
- A. E. Taylor and W. R. Mann, *Advanced Calculus*, Wiley, 1983
- M. do Carmo, *Differential Geometry of Curves and Surfaces*, Birkhäuser, 1992

General description:

Broadly speaking, "differential geometry" is the use of tools of calculus to study problems in geometry. More specifically, in this course, we will apply tools of multivariable and vector calculus to study geometric properties of curves and surfaces in 3-space. Whereas topology, the subject matter of Math 441, is concerned with properties that are preserved by homeomorphisms (continuous bijections with continuous inverses), geometry is concerned with properties that are preserved by distance-preserving homeomorphisms. Such properties include distances (of course) and angles, as well as other familiar non-topological properties such as lengths of curves, areas, and volumes, all of which require calculus to compute in general. And the most important geometric property of all is one that cannot even be defined without calculus—curvature.

Specifically, in 442 and 443 we will cover most or all of the following topics: the theory of curves in the plane and in 3-space, the extrinsic local theory of surfaces in 3-space (looking at a surface from outside), the intrinsic local theory of surfaces (looking at a surface from inside), geodesics, curvature, spherical and hyperbolic geometry, and the global theory of surfaces, culminating in the Gauss–Bonnet theorem.

Because of the focus on calculus, the flavor of the course will be very different from that of 441. It will be in some ways less abstract than 441, but because we will introduce quite a few new technical tools, you will need to learn at least as many new definitions, concepts, and techniques as in 441. The payoff is that you should end up having mastered the tools needed to delve into one of the most beautiful and physically relevant branches of modern mathematics.

Prerequisites:

The official prerequisite is grades of 2.0 or higher in Math 308, 326, 328, and 441. However, grades of 3.0 or higher in all of these courses would be more realistic. In addition, the most important prerequisite is a genuine interest in abstract mathematics, or at least a mind open to the possibility. Intellectual curiosity and an ability to express mathematics clearly in writing are also important. Of course, one purpose of the course is to improve such skills.

Specific prerequisite material includes the following. If two or three items on the list are unfamiliar, you are probably still OK as long as you're willing to spend some extra time reviewing. If there are half a dozen unfamiliar items on the list, you're not ready for this course. Of course many people will be rusty on various topics. Some of these topics will be reviewed briefly in class as they arise, but be prepared to do some review on your own.

- Vector calculus: partial derivatives, the chain rule, dot products, cross products, tangent lines, tangent planes, line integrals, surface integrals, gradients, vector fields, the divergence theorem.
- Linear algebra: vector spaces, bases and dimension, linear transformations and their representation by matrices, determinants, matrix algebra, eigenvectors and eigenvalues, inner products.
- Analysis in several variables: differentials, Jacobians, the inverse and implicit function theorems.
- Topology: open and closed sets, boundaries, limit points, closures, continuous maps, homeomorphisms, connected sets, compact sets.

Homework:

A homework assignment will be given out each week, due a week later. Plan to spend a lot of time on homework—six hours or more outside of class during most weeks.

A typical homework assignment will consist of the following:

I. Reading: Typically, you will be given one or two sections of the textbook to read each week. I'll expect you to read through the chapter quickly before the relevant lectures, and then to reread it carefully after the lectures.

II. Practice Problems: Most assignments will include a number of "practice problems." These are not to be handed in for a grade, but I expect you to do (or at least figure out how to do) all of them for your own good. Understanding these problems will be important for solidifying your understanding of the text and lectures, and for preparing to do the required problems. Some of these problems are likely to show up on exams.

III. Required Problems: The problems listed as "Required Problems" are for you to write up and hand in for a grade. These problems, consisting almost entirely of proofs, are the heart of the course, and they will constitute a significant part of your course grade.

Here are some other important instructions regarding homework:

- **Collaboration:** I strongly encourage you to work with other students on the homework. Discussing problems and ideas with your classmates is one of the best ways to learn the material. But when writing up solutions to hand in, please *write your own solutions in your own words.*
- **Citing results:** You may freely cite theorems, etc., as well as results of exercises, from earlier in the book. If you do use a previous result, be sure you identify it clearly: by giving its name, or its theorem number, or stating it correctly.
- **Assembly:** Arrange your solutions in numerical order, just as they appear on the assignment sheet, with each problem starting on a new page. Problems that are out of order might not get credit. Please staple the pages of each assignment together.
- **Identification:** Make sure the first page of each homework packet is clearly labeled with your name and the assignment number.
- **Typesetting vs. handwriting:** If you are comfortable doing so, I encourage you to submit computer-typeset assignments. I highly recommend LaTeX, since that is the de facto standard in mathematics; but any typesetting program will do. I've posted some helpful typesetting links on the class web page. I'm also happy to accept handwritten assignments, as long as they are neat and legible (see below).
- **Legibility:** If you write by hand, write your answers neatly and legibly, not too small, with as few erasures or crossouts as possible. Be sure to distinguish clearly between similar symbols, such as a/α, $b/6$, $C/\subset$, $\in/\varepsilon$, $g/q/9$, h/n, $I/l/1$, p/ρ, r/γ, $s/5$, $t/+$, $u/v/\nu$, $U/\cup$, $x/\times/\chi$, $y/4$, $z/2$, ζ/ξ, and uppercase/lowercase letters. Unless mathematical ideas spring fully and impeccably realized from your pen, your first draft is not acceptable.
- **White space:** Don't be stingy with white space. *Leave one-inch margins on all sides of your pages.*

Exams:

- Midterm: tentatively scheduled for Monday, February 7.
- Final: Thursday, March 17, 8:30-10:20.

During each exam, you may use two $8\frac{1}{2}'' \times 11''$ one-sided pages (or one sheet written on both sides) of your own handwritten notes. No photocopied or printed material is allowed. You may not share notes with other students.

Grading:

Before computing your final grade, I'll convert each homework score to a percentage, so that each assignment carries equal weight. Then your lowest homework score will be dropped, and the remaining assignments will be averaged.

Your grade will be based on a weighted average of the following scores:

- homework (30%)
- midterm (25%)
- final exam (45%)