Math 544/545/546 | **Topology and Geometry of Manifolds** | **2019–2020**

SYLLABUS

Course Web site: www.math.washington.edu/~lee/Courses/544-2019

Lectures: MWF 1:30–2:20
Padelford C-036

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Required Textbooks:
For Math 544: *Introduction to Topological Manifolds*, 2nd edition, by John M. Lee [ITM]
For Math 545–546: *Introduction to Smooth Manifolds*, 2nd edition, by John M. Lee [ISM]

UW students can download free PDF copies of both texts for your personal use from the UW Libraries website. If you purchase a printed copy and show it to me with your name written in ink, I'll reimburse you for my share of the list price ($3.75 for a paperback, $11.25 for the hardcover version of [ITM], $14.85 for the hardcover version of [ISM]). Be sure to download the latest list of corrections to the text (linked from the course web page), and mark them in your copy.

General description:
The three-quarter sequence Math 544-545-546 is a graduate-level introduction to topology and differential geometry, with primary focus on *manifolds*. These are arbitrary-dimensional generalizations of curves and surfaces—spaces that locally look like Euclidean space but globally may not, just as a sphere looks like a plane if you zoom in far enough, but is globally very different. They are the basic subject matter of differential geometry, but also play a role in many other branches of pure and applied mathematics.

In Math 544 (fall quarter) we will concentrate on the *topology* of manifolds, i.e., properties that are invariant under continuous deformations. The main goals are to understand the fundamental group, covering spaces, and the classification of compact surfaces. Along the way, we will take the opportunity to study topology in somewhat greater generality, not just on manifolds. We'll cover virtually all of Chapters 1–12 of [ITM].

Then Math 545 and 546 (winter and spring quarters) will be devoted to the study of *smooth manifolds*, on which derivatives of functions and maps make sense. We will study the basic flora and fauna that live on them: tangent vectors and covectors, submanifolds, vector and tensor fields, flows, Lie derivatives, Lie groups, Riemannian metrics, differential forms, orientations, de Rham cohomology, foliations, and group actions. (Our treatment of Riemannian metrics will be pretty brief; a deep examination of the geometric properties of Riemannian metrics will have to wait until the sequel to this course, Math 547.) These two quarters will cover most of Chapters 1–16 and 18–21 of [ISM].

Religious Accommodations:
Washington state law requires that UW develop a policy for accommodation of student absences or significant hardship due to reasons of faith or conscience, or for organized religious activities. The UW's policy, including more information about how to request an accommodation, is available at *Religious Accommodations Policy* (https://registrar.washington.edu/staffandfaculty/religious-accommodations-policy/). Accommodations must be requested within the first two weeks of this course using the *Religious Accommodations Request form* (https://registrar.washington.edu/students/religious-accommodations-request/).

Other Accommodations:
If you need disability accommodations, please make arrangements through Disability Resources for Students (depts.washington.edu/uwdrs). If you must turn in an assignment late due to medical or other compelling reasons, contact me for permission in advance, or as soon as medically possible thereafter.

Prerequisites:

The prerequisites for the manifolds sequence are listed below. Note that this list encompasses much of the mathematics that you've (hopefully) been exposed to in your undergraduate math courses. For this reason, many first-year grad students find the manifolds sequence a bit harder than the algebra or analysis sequence. Whereas graduate-level algebra and analysis are basically continuations of the corresponding undergraduate courses, the study of manifolds draws on ideas from a lot of different areas of mathematics and therefore often feels quite abstract and exotic. If you aren't familiar with one or two of these topics, you should fill in the gaps by independent reading. But if you find a lot of items on the list unfamiliar, you should consider taking a lower-level course that covers some of these prerequisites. See me if you have questions about your preparation for the course.

For Fall quarter

- *Set Theory*: Operations on sets, functions, equivalence and order relations, number systems and cardinality, the axiom of choice. References: [R, Chapter 1]; [A, Chapters 1 & 2]; [ITM, Appendix A].
- *Analysis*: Metric spaces; convergence and continuity; open and closed subsets; interior, exterior, and boundary; compactness. References: [R, Chapters 2–4]; [A, Chapters 3 and 4]; [ITM, Appendix B].
- *Algebra*: Elementary group theory, homomorphisms, isomorphisms, subgroups, normal subgroups, permutation groups, cyclic groups, cosets, quotient groups. References: [Hu, Chapter 7]; [He, Chapters 1–3]; [ITM, Appendix C].

For Winter and Spring quarters

- *Topology*: All the material covered in Math 544. Reference: [ITM]; [ISM, Appendix A].
- *Linear algebra*: Abstract vector spaces, subspaces, bases, dimension, matrices, determinants, change of basis formulas, linear maps, kernel & image, norms & inner products, orthonormal bases. References: any rigorous linear algebra text, such as [FIS]; [ISM, Appendix B].
- *Multivariable calculus*: Partial derivatives; the total derivative as a linear approximation; Taylor's formula in several variables; multiple integrals and the change of variables formula; gradient, divergence, and curl; the theorems of Green, Gauss, and Stokes; uniform convergence. References: [MTW]; [R, Chapters 5–7]; [A, Chapters 12–14]; [ISM, Appendix C].
- *Differential equations*: Basic facts about existence and uniqueness of solutions to ODEs; elementary techniques for solving first-order equations and systems at the level of Math 307 and 309. Reference: [BDM]; [ISM, Appendix D].

References

[A] Tom Apostol, *Mathematical Analysis*, 2nd edition, Addison–Wesley, 1974.

[BDM] William E. Boyce, Richard C. DiPrima, and Douglas B. Meade, *Elementary Differential Equations and Boundary Value Problems*, 11th edition, Wiley, 2017.

[FIS] Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence, *Linear Algebra*, 4th edition, Pearson, 2003.

[He] Israel N. Herstein, *Abstract Algebra*, 3rd edition, Wiley, 1996.

[Hu] Thomas W. Hungerford, *Abstract Algebra: An Introduction*, 3rd edition, Cengage, 2013.

[ITM] John M. Lee, *Introduction to Topological Manifolds*, 2nd edition, Springer, 2011.

[ISM] John M. Lee, *Introduction to Smooth Manifolds*, 2nd edition, Springer, 2013.

[MTW] Jerrold E. Marsden, Anthony Tromba, and Alan Weinstein, *Basic Multivariable Calculus*, Springer, 1993.

[R] Walter Rudin, *Principles of Mathematical Analysis*, 3rd edition, McGraw–Hill, 1976.

Requirements:

Here are the things I'll expect you to do for this course.

- **Lectures:** Although I won't be taking attendance, I expect you to attend all lectures. I don't intend to spend a lot of time repeating details that are in the books; instead, lectures will be a time for me to offer insights and suggestions about how to think about the definitions, proofs, and homework problems, and to answer any questions that you may have. (Please don't hesitate to interrupt me to ask questions. If you're wondering about the meaning of something I said, or why something is true, it's likely that several others in the class are also wondering. If I think you're interrupting too much – which is unlikely – I'll suggest that we continue the discussion outside of class.)
- **Reading:** Typically, you'll be given approximately one chapter to read each week. The reading is absolutely central to this course: I've taken pains to make sure that the textbooks contain all of the essential ideas that you need to master, together with detailed definitions and proofs, and a lot of motivating intuition. You need to really *read* the assignments – don't expect to be able to look at the homework problems and then leaf back through the chapter to find examples of how to do those kinds of problems. My suggestion is that you skim through each chapter quickly before the relevant lectures, and then reread it carefully after the lectures.
- **Exercises:** Scattered throughout the textbooks are various *Exercises* (as distinct from the *Problems* at the ends of the chapters). I expect you to do (or at least figure out how to do) *all of them* as you read. Most of these are relatively routine, and will not be collected or graded, but nevertheless understanding them is important to the continuity of the text and the lectures.
- **Things to work out on your own:** I may post some Problems that you should work out for your own educational benefit, in addition to the Exercises. They might present facts or examples that you should know about, or results that might be useful in later Problems.
- **Written homework:** Each week, I'll post a written homework assignment, due one week later. These problems are the heart of the course. See the next page for guidelines on writing them up.
- **Exams:** In the fall quarter, there will be a take-home midterm exam and a take-home final exam, both cumulative. The exact dates will be announced later. The exam schedules for winter and spring quarters are still up in the air, and will be announced at the beginning of each quarter.

Learning Objectives:

To become familiar with the principal definitions, tools, and theorems about topological and smooth manifolds; to develop geometric intuition about manifolds and learn to make it rigorous; to become skillful with the proof techniques used in topology and differential geometry; to prepare for future study in differential geometry, algebraic topology, algebraic geometry, or global analysis; to learn about the many applications of manifolds in other fields of mathematics; to prepare for the Manifolds preliminary exam.

Grading:

Your 544 grade will be based on a weighted average of homework and exam scores:

- 50% written homework problems
- 15% midterm exam
- 35% final exam

I won't decide on the exact grading scale until the end of the quarter, but when I've taught this course in past years an average of about 90% yielded a 4.0 and 70% a 3.0, with other scores linearly interpolated or extrapolated. The minimum grade that can be counted toward a graduate degree in mathematics is 3.0.

Homework guidelines:

When you write up homework assignments to hand in, please follow these guidelines:

- **Collaboration:** I strongly encourage you to work with other Math 544/5/6 students on the homework. You'll get the most benefit from working with others if you make a good-faith effort to solve the problems on your own first; but once you've thought about them for a while, you'll learn a lot from discussing them with other classmates. When writing up solutions to hand in, *you must write your own solutions in your own words.* Don't look at anyone's written solutions (including other students' homework papers, published proofs, or solutions posted on the internet) before turning in your homework. Any unattributed use of material from any written source, including the internet, constitutes plagiarism.
- **Problem statements:** For each written homework problem, please include a clear statement of what you're proving. You need not copy the entire problem statement, but be sure to include enough that the grader can tell exactly what you're claiming to prove. I prefer that you state each result in the form of a theorem (e.g., "Theorem: Every closed subspace of a paracompact space is paracompact") instead of a command ("Prove that every closed subspace of a paracompact space is paracompact") or a question ("Is every closed subspace of a paracompact space paracompact?").
- **Citing results:** You may freely use the results of Exercises, Theorems, Propositions, Corollaries, and Lemmas from earlier in the book. (For this purpose, consider the appendices to be earlier than all the other chapters.) Unless otherwise stated, you may only use another Problem if it has been previously assigned (either to work out on your own or to hand in), or if you give its solution. Results from other books or the internet can be used only if you prove them, or if they are part of the undergraduate prerequisites listed on page 2.
- **Proof-writing conventions:** On the class web page, there's a link to a short note describing standard conventions for writing mathematical proofs. Learn them and follow them! The sooner you get used to following them, the easier it will be for you to develop effective mathematical writing habits.
- **Typesetting vs. handwriting:** I encourage you to submit computer-typeset assignments. I recommend LaTeX, since it's the de facto standard in mathematics, and you'll have to learn it sooner or later if you continue doing math research or teaching; but any typesetting program will do. On the class website, there's a link to some helpful references about mathematical typesetting. If you do typeset your assignments, please print them out and hand in hardcopies. I'm also happy to accept handwritten assignments, as long as they're neat and legible (see below).
- **Legibility:** If you write by hand, write your answers neatly and legibly, not too small, with as few erasures or crossouts as possible. Be sure to distinguish clearly between similar symbols, such as $l/1$, $b/6$, $\in/\varepsilon$, $g/q/9$, h/n, p/ρ, r/γ, $s/5$, $t/+$, v/ν, $x/\times$, $y/4$, $z/2$, $\subset/C$, $\cup/U$, and uppercase/lowercase letters. Unless mathematical ideas spring fully and impeccably realized from your pen, your first draft is not acceptable.
- **Assembly:** Arrange your solutions in numerical order, just as they appear on the assignment page, with each problem starting on a new page. Problems that are out of order might not get credit. Please staple the pages of each assignment together.
- **Identification:** Make sure the first page of each homework packet is clearly labeled with your name and the assignment number.
- **White space:** Don't be stingy with white space. *Leave one-inch margins on all four edges of your pages.* If you don't, the grader will be annoyed because he doesn't have room to write comments, and you don't want your paper being evaluated by an annoyed grader!
- **Due date:** Each homework assignment must be turned in within the first ten minutes of class on the due date. After the first ten minutes, it will receive a deduction for lateness. Assignments turned in after class won't be accepted unless you get permission from me in advance. (The reason for the strictness of this rule is to dissuade you from skipping class in order to finish the homework.)