Math 549 | **Geometric Structures** | **Spring 2004**

Complex Manifolds

COURSE DESCRIPTION

Lectures: MWF 1:30–2:20
Padelford C-401

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Office Hours: Wed. & Fri. 2:30–3:20 or by appointment

Web: `www.math.washington.edu/~lee/Courses/549-2004`
(or from the Math Department home page,
Selected Course Web Pages → Math 549)

Books:

There will be no required textbook for this course. Attached is a list of suggested references on complex manifolds and background material that you can consult if you wish.

Prerequisites:

- *Topology* at the level of Math 544. Reference: [ITM].
- *Smooth manifolds* at the level of Math 545/546. Reference: [ISM].
- *Riemannian geometry* at the level of Math 547. Reference: [RM]
- *Complex analysis* at the level of Math 534. References: [Ah] or [Co].

(The references in brackets refer to the second page of the suggested reading list.) Some acquaintance with homology theory and/or elliptic partial differential equations would be helpful, but I will not assume any prior knowledge of these subjects. (Chapter 13 of [ITM] covers all of the homology theory you'll need.)

Homework and grades:

I will assign homework problems on an irregular basis. If you register for a grade, your grade will be based on how many of the assigned homework problems you do correctly. Roughly speaking, 75% will get you a 4.0, and 25% a 3.0, linearly interpolated in between. You may also choose to register for S/NS grading; the requirement for an S grade is regular attendance and participation in class discussions. (Note that Mathematics graduate students who have not yet reached *precandidate* status are required to register for three graded courses each quarter.)

Course outline:

I hope to cover as many of the following topics as time permits, in roughly this order:

- Definition and examples of complex manifolds
- Almost complex structures and integrability
- Complex vector bundles
- Sheaves and cohomology
- Line bundles and divisors
- Hermitian and Kähler metrics
- Connections and Chern classes
- Hodge theory
- Kähler-Einstein metrics and Calabi-Yau manifolds
- The Kodaira embedding theorem

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SUGGESTED READING

Books about complex manifolds

With the exception of [M], which can be downloaded from the Internet, these books are (or soon should be) on reserve in the Math Research Library.

[C] S.-S. Chern, *Complex Manifolds Without Potential Theory*, New York, Springer-Verlag, 1979. A straightforward introduction to most of the material we will study, although it's a little hard to read because of the pre-TEX typography. The appendix gives an excellent treatment of the Chern-Weil theory of characteristic classes, straight from the horse's mouth. (The odd title refers to the fact that the book does not treat Hodge theory. Classically, the term "potential theory" refers to the study of Laplace's equation; since Hodge theory is about solutions to the Laplace-Beltrami operator, which is a differential-form analogue of the Laplacian, Hodge theory can be loosely considered as a branch of potential theory.)

[GH] P. Griffiths and J. Harris, *Principles of Algebraic Geometry*, New York, John Wiley & Sons, 1978. Despite the title, this beautiful book is really for the most part about complex manifolds, with particular emphasis on those that arise as projective algebraic varieties. This book's treatment of the subject is probably closest to the one I will follow.

[G] R. C. Gunning, *Lectures on Riemann Surfaces*, Princeton, Princeton University Press, 1966. An excellent treatment of Riemann surfaces (one-dimensional complex manifolds). Also, §§2–3 give a treatment of sheaves and sheaf cohomology very similar to the one I will give.

[KN] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry*, New York, Interscience Publishers, 1969. This two-volume tome is an encyclopedic reference for real and complex differential geometry, with a thorough treatment of topics such as bundles, connections, holonomy, and characteristic classes. It's hard to read if you don't already have a basic understanding of the material, but it serves as a good "reference manual." Chapter IX of Volume 2 covers complex manifolds and Kähler metrics.

[M] A. Moroianu, *Lectures on Kähler Geometry*, `www.arxiv.org/math.DG/0402223`. This is a brand-new set of lecture notes designed for a graduate course on Kähler metrics, which will be a major topic in the latter part of this course. I can't vouch for their correctness or readability, but on first reading they look very good.

[MK] J. Morrow and K. Kodaira, *Complex Manifolds*, New York, Holt Rinehart and Winston, 1971. The main purpose of this book is to give an introduction to Kodaira's theory of deformations of complex structures, written by our own Jim Morrow when he was Kodaira's student; but it starts out with a nice introduction to general complex manifold theory.

[W] R. O. Wells, Jr., *Differential Analysis on Complex Manifolds*, New York, Springer-Verlag, 1980. This book is nicely written, although some of the material is presented in a rather idiosyncratic way. It is most useful for its excellent treatment of elliptic operator theory and the Hodge theorem.

[Z] F. Zheng, *Complex Differential Geometry*, Providence, American Mathematical Society, 2000. This is a fairly new book that I haven't had a chance to read yet. But its table of contents suggests that it covers much of the same material that I'll cover in this course.

Background and related material

[Ah] L. Ahlfors, *Complex Analysis*, New York, McGraw-Hill, 1979.

[Co] J. B. Conway, *Functions of One Complex Variable I*, New York, Springer-Verlag, 1995.

[ITM] J. M. Lee, *Introduction to Topological Manifolds*, New York, Springer-Verlag, 2000.

[ISM] J. M. Lee, *Introduction to Smooth Manifolds*, New York, Springer-Verlag, 2003.

[RM] J. M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, New York, Springer-Verlag, 1997.

[K] S. G. Krantz, *Function theory of several complex variables*, Pacific Grove, CA, Brooks/Cole, 1992.