Math 120 - Fall 2021
Final Exam

NAME (First,Last) : ....

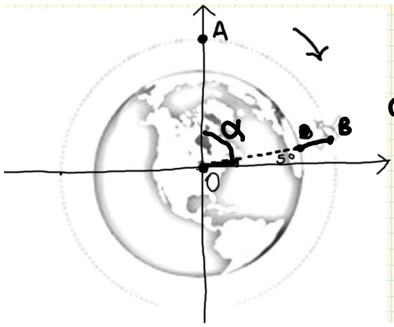
UW email: ....

Student ID ....

- you here 2.5 hours to complete this exam.
- Please use the same name that appears in Canvas.
- Make sure your writing is clear and dark enough.
- If you run out of space, continue your work on the last (blank) page and indicate clearly on the problem page that you have done so.
- Unless stated otherwise, you **MUST** show work for credit.
- Your work needs to be neat and legible.
- Unless the problem gives you different instructions, you can give exact answers or round off your answers to 2 decimal places.
- The only calculator allowed is the TI 30X IIS. You are allowed an 8x11 sheet of notes, written both sides.
- Box your final answer, when appropriate.
- Raise your hand if you have a question.

10 pt.

1. A satellite is circling Earth, moving in the direction indicated by the arrow (see picture below) . At 6 PM on 12/10/2021 it passes by point A, and 40 minutes later the satellite is at B. The angle the line OB forms with the horizontal is 5°.



$$\alpha = \frac{\pi}{2} - \frac{5 \cdot 2\pi}{360} = \omega \cdot 40$$

$$\frac{18\pi}{36} - \frac{\pi}{36} = \omega \cdot 40$$

$$\frac{17\pi}{36 \cdot 40} = \omega \quad \text{red/min}$$

(a) How many minutes does it take the satellite to complete a full revolution ?

$$T = \frac{2\pi}{\omega} = \frac{2\cancel{\pi} \cdot 36 \cdot 40}{17\cancel{\pi}} = 169.41 \text{ min}$$

4 pt

(b) The satellite is orbiting at a distance of 7000 miles from the center of the earth. Impose a coordinate system with the origin at the center of the earth, like in the picture above, and find the coordinates of the satellite at time t minutes after 6 PM on 12/10/202[redacted].

$$x = 7000 \cos\left(-\frac{17\pi}{1440} t + \frac{\pi}{2}\right)$$

$$y = 7000 \sin\left(-\frac{17\pi}{1440} t + \frac{\pi}{2}\right)$$

6 pt

1pt $r = 7000$

2pt $\omega = -\frac{17\pi}{36 \cdot 40}$

2pt $\varphi = \frac{\pi}{2}$

1 pt cos/sin

15 pt

2. An infectious disease is spreading in SmallTown in ~~2020~~ 2019. The number of people testing positive on any given day can be modeled by a sinusoidal function. On February 15, ~~2020~~ 2019 (the 46^{th} day of the year) there was a peak (highest number) of 52 positive tests; after that the daily number of positive tests kept decreasing and reached a minimum (lowest number) of 10 on April 4, ~~2020~~ 2019 (the 94^{th} day of the year).

   (a) According to this sinusoidal model, how many people tested positive on ~~January 1,2020~~ Dec 31^{st} 2019 (the ~~1^{st}~~ 365^{th} day of the year)? (Round to the closest integer).

$$f(t) = \overbrace{A \sin\left(\frac{2\pi}{B}(t-c)\right) + D}^{4\text{ pt}} \qquad \# \text{ positive on day } t$$

$$B = 2(94-46) = 96$$

$$A = \frac{52-10}{2} = 21$$

$$D = \frac{52+10}{2} = 31$$

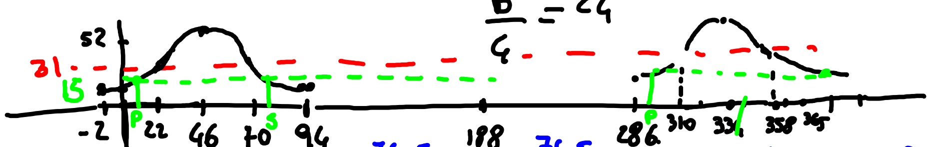
$$C = 46 - \frac{96}{4} = 22$$

$$f(t) = 21 \sin\left(\frac{2\pi}{96}(t-22)\right) + 31$$

$$f(365) \approx 22 \quad 2\text{pt}$$

   (b) For how many days in 2020 was the daily number of positive tests at least 15 ? (Round to the closest integer).

$$\frac{B}{4} = 24$$



$$365 - (8.76 + 3 \cdot 96) = 68.24$$

$$21 \sin\left(\frac{2\pi}{96}(t-22)\right) + 31 = 15 \quad 2\text{pt}$$

$$\sin\left(\frac{2\pi}{96}(t-22)\right) = -\frac{16}{21}$$

$$t = \frac{96}{2\pi} \arcsin\left(-\frac{16}{21}\right) + 22 \approx 8.76 \quad 2\text{pt} \qquad \text{principal sol}$$

$$46 + (46 - 8.76) = 83.26 \quad 2\text{pt} \qquad \text{Symmetric sol}$$

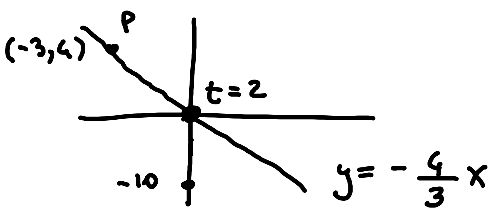
In one period: $83.26 - 8.76 \approx 74.5$ days

Total $74.5 \times 3 + (365 - (8.76 + 3 \times 96)) \approx 292$ days 3 pt

15 pt

3. At 8:00 AM one day Steve starts walking from the point (-3,4) on the line $y = -\frac{4}{3}x$ in the SE direction (i. e. towards the origin). He walks at a constant speed of 2.5 mph. Elsie ~~walks~~ starts walking North along the y axis ~~staring~~ at 10:00 am from the point (0,-10) ~~at 10:00 AM~~. Elsie walks at 2 mph. Let t=0 correspond to 8:00 am.

   (a) Find the parametric equations for Steve.



$d(P,O) = \sqrt{3^2+4^2} = 5$

$t = \frac{5}{2.5} = 2$ Steve reaches origin at $t=2$

$x = -3 + \frac{0-(-3)}{2}t = -3 + 1.5t$ 2pt

$y = 4 + \frac{0-4}{2}t = 4 - 2t$ 2pt

   (b) Find the parametric equations for Elsie.

$x = 0$ 1 pt

$y = -10 + 2(t-2)$ 2pt

   (c) When are Steve and Elsie closest to each other ? (Assume they keep walking forever). Give your answer as a time of day in the normal HH:MM AM/PM way, for example, 7:30 AM.

we want to minimize the distance between Steve and Elsie; it is ok to minimize d^2

$d^2(t) = (-3+1.5t)^2 + (4-2t-(-10+2(t-2)))^2$ 2pt

$9 - 9t + 2.25t^2 + (4-2t+10-2t+4)^2$

$9 - 9t + 2.25t^2 + (18-4t)^2$ 2pt

$9 - 9t + 2.25t^2 + 18^2 + 16t^2 - 144t = 18.25t^2 - 153t + 333$

$\cup$ has a min at $\frac{153}{18.25 \times 2} = 4.19178$; $0.19178 \times 60 = 11.51$ 3pt

at 12:12 pm 1pt

10 pt

4. Consider the function $f(x) = x^2 - 8x + 15$ defined on $-\infty < x < \infty$

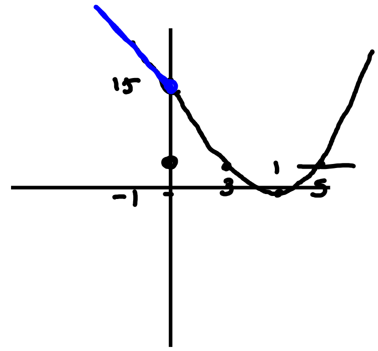
(a) The x intercepts of this functions are at x=3 and x=5. Draw the graph of $y = f(x)$. Mark and show the value of all x and y intercepts and of the vertex.

$h = \frac{8}{2} = 4$

$k = 16 - 32 + 15 = -1$

V at $(4, -1)$

3 pt



(b) Restrict f to the domain $x \leq 0$ and compute $f^{-1}(x)$, the inverse of f.

$y = x^2 - 8x + 15$

$x^2 - 8x + 15 - y = 0$

$x = \frac{8 \pm \sqrt{64 - 4(15-y)}}{2}$ $\quad x < 0 \quad$ so

$x = 4 - \frac{\sqrt{4 + 4y}}{2} = 4 - \sqrt{1 + y}$

$f^{-1}(x) = 4 - \sqrt{1+x}$ $\quad$ 3 pt

(c) What are the domain and range of $f^{-1}(x)$?
Domain:
Range:

$[15, +\infty)$ $\quad$ 2pt

$(-\infty \quad 0]$ $\quad$ 2pt

10 pt

5. The number of oak trees in Oakville doubles every 30 years; the number of Maples in Nomapletown decreases 10% every 5 years. Last year the number of oak trees in Oakville was the same as the number of maple trees in Nomapletown. In how many years (from this year) will the number of Maples in Nomapletown be half of the number of oaks in Oakville?

$t = 0$ last year

$f(t) = A\, \sqrt[30]{2}^{\,t}$ # oak trees 3 pt

$g(t) = A\, \sqrt[5]{0.9}^{\,t}$ # maples 3 pt

Want $g(t) = \frac{1}{2} f(t)$ 2 pt

$$\cancel{A}\, \sqrt[5]{0.9}^{\,t} = \frac{1}{2} \cancel{A}\, \sqrt[30]{2}^{\,t}$$

$$t \ln \sqrt[5]{0.9} = \ln \frac{1}{2} + t \ln \sqrt[30]{2}$$

$$t = \frac{\ln \frac{1}{2}}{\ln \sqrt[5]{0.9} - \ln \sqrt[30]{2}} \approx 15.69$$ 1 pt years from last year

14.69 years from this year

1 pt