Math 120 - Winter 2018
Final Exam
March 10th, 2018

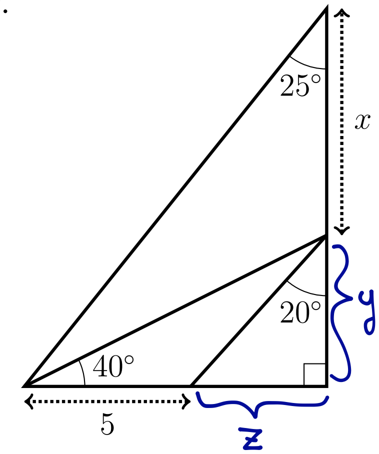
Name: ______________________ Student ID no. : ______________

Signature: ______________________________ Section: ____________

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	70	

- This exam consists of SEVEN problems on FIVE double-sided pages.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic devices are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- $\boxed{\text{Draw a box}}$ around your final answer to each problem.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the back of the first or last page and indicate that you have done so. If you still need more room, ask us for an extra page to staple to your exam.
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 170 minutes to complete the exam.

1. **[10 points]** In the following figure (not drawn to scale), find x.



$$\tan(20^\circ) = \frac{z}{y} \longrightarrow z = y\tan(20^\circ)$$

$$\tan(40^\circ) = \frac{y}{z+5} \longrightarrow y = (y\tan(20^\circ) + 5)\tan(40^\circ)$$

$$\tan(25^\circ) = \frac{5+z}{x+y}$$

$$y - y\tan(20^\circ)\tan(40^\circ) = 5\tan(40^\circ)$$

$$y = \frac{5\tan(40^\circ)}{1 - \tan(20^\circ)\tan(40^\circ)}$$

$$z = \frac{5\tan(20^\circ)\tan(40^\circ)}{\underbrace{1 - \tan(20^\circ)\tan(40^\circ)}}$$

$$x + y = \frac{5+z}{\tan(25^\circ)}$$

$$x = \frac{5+}{\tan(25^\circ)} - \frac{5\tan(40^\circ)}{1 - \tan(20^\circ)\tan(40^\circ)} \approx \boxed{9.397}$$

2. Polya and Baxter begin running around a circular racetrack at the same time.

Polya starts at the northernmost point of the track and runs clockwise at a speed of 6 meters per second. It takes him 50 seconds to run one complete lap.

Baxter runs counterclockwise at a speed of 4 meters per second.

Polya and Baxter first pass each other after 10 seconds.

(a) **[3 points]** Find Baxter's angular speed in radians per second.

Polya:

$v=6$

$\omega = \frac{2\pi}{50} = \frac{\pi}{25}$

$r = \frac{v}{\omega} = \frac{150}{\pi}$

Baxter:

$v=4$

$r = \frac{150}{\pi}$

$\omega = \frac{v}{r} = \boxed{\frac{2\pi}{75}}$

(b) **[7 points]** Find Baxter's coordinates after two minutes.

(Place the origin at the center of the track, and measure units in meters.)

$x_0 = 0 \quad y_0 = 0 \quad r = \frac{150}{\pi} \quad \omega = \frac{2\pi}{75}$

$\theta_0 + 10\left(\frac{2\pi}{75}\right) = \frac{\pi}{2} - 10\left(\frac{\pi}{25}\right)$

$\theta_0 = \frac{\pi}{2} - \frac{2\pi}{5} - \frac{4\pi}{15} = \frac{-\pi}{6}$

$x = \frac{150}{\pi}\cos\left(\frac{-\pi}{6} + 120\left(\frac{2\pi}{75}\right)\right) \approx -47.485$

$y = \frac{150}{\pi}\sin\left(\frac{-\pi}{6} + 120\left(\frac{2\pi}{75}\right)\right) \approx -4.991$

$\boxed{(-47.485, -4.991)}$

3. The tides in Sea-attle are a sinusoidal function of time.

The first high tide today was at 3:00 AM, with a depth of 5 meters above low tide.

The first low tide was at 6:30 AM.

(a) **[4 points]** Write a function $y = f(t)$ for the water depth today, t hours after midnight. Let $y = 0$ be low tide.

$$f(t) = A\sin\left(\frac{2\pi}{B}(t-C)\right) + D$$

$A = 2.5$ $\quad C = 1.25$

$B = 7$ $\quad D = 2.5$

$$\boxed{f(t) = 2.5\sin\left(\frac{2\pi}{7}(t-1.25)\right) + 2.5}$$

(b) **[6 points]** Today (from midnight to midnight), for how much time is the water at a depth greater than 3 meters above low tide?

$$2.5\sin\left(\frac{2\pi}{7}(t-1.25)\right) + 2.5 = 3$$

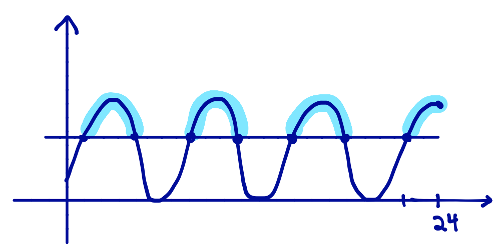
$$\sin\left(\frac{2\pi}{7}(t-1.25)\right) = \frac{1}{5}$$

$$\frac{2\pi}{7}(t-1.25) = \sin^{-1}\left(\frac{1}{5}\right)$$

principal sol'n.

$$t = 1.25 + \frac{7}{2\pi}\sin^{-1}\left(\frac{1}{5}\right) \approx 1.474$$

symmetry solution: $2(1.25) + 3.5 - p \approx 4.526$



list of solutions:

1.474
4.526
8.474
11.526
15.474
18.526
22.474

+7 +7 +7 +7 +7

Total time: $(4.526 - 1.474) \cdot 3.5 = \boxed{10.680 \text{ hours}}$

4. Sea-attle is also full of marine life.

(a) **[3 points]** The number of flounders in Sea-attle doubles every 24 years.

In the year 2000, there were 7500 flounders in Sea-attle.

Write a function $f(t)$ for the number of flounders in Sea-attle, t years after 2000.

$f(t) = A_0 b^t$

$A_0 = 7500$

$b = 2^{1/24}$

$$\boxed{f(t) = 7500\left(2^{1/24}\right)^t}$$

(b) **[3 points]** The number of groupers in Sea-attle increases by 4.2% per year.

In the year 2018, there were 2 groupers per flounder.

Write a function $g(t)$ for the number of groupers in Sea-attle, t years after 2000.

$g(t) = A_0 b^t$

$b = 1.042$

$A_0(1.042)^{18} = 2f(18) = 2 \cdot 7500 \cdot 2^{18/24}$

$A_0 = \dfrac{15000 \cdot 2^{3/4}}{1.042^{18}} \approx 12029$

$$\boxed{g(t) = 12029 \cdot 1.042^t}$$

(c) **[4 points]** In what year will there be 3 groupers per flounder?

$12029 \cdot 1.042^t = 3 \cdot 7500 \cdot \left(2^{1/24}\right)^t$

$\dfrac{12029}{22500} \; 1.042^t = \left(2^{1/24}\right)^t$

$\ln\left(\dfrac{12029}{22500}\right) + t\ln(1.042) = t\ln\left(2^{1/24}\right)$

$t = \dfrac{\ln\left(\frac{12029}{25000}\right)}{\ln\left(2^{1/24}\right) - \ln(1.042)} \approx 51.07$, so around $\boxed{2051}$

5. I'm starting a business selling flounder-and-grouper sandwiches.

The number of sandwiches I'll sell per day is a linear function of the price.

If I charge $\$3$, I'll sell 99 sandwiches per day.

If I charge $\$7$, I'll sell 39 sandwiches per day.

(a) **[4 points]** Write a function $f(x)$ for the number of sandwiches I'll sell per day if I charge $\$x$ per sandwich.

slope: $\frac{-60}{4} = -15$

$f(x) = -15(x-3)+99$

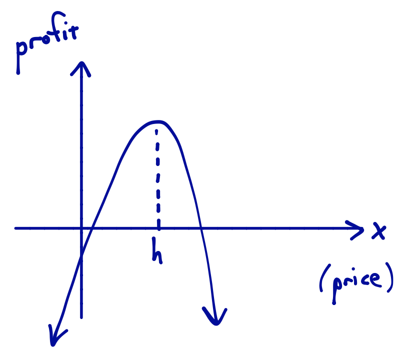
$\boxed{f(x) = -15x + 144}$

(b) **[6 points]** Suppose it costs me $\$2$ to make each sandwich.

How much should I charge to maximize my daily profit?

(Remember: profit is the amount of money I earn from selling sandwiches, minus the cost of making those sandwiches.)

$$\text{Profit} = \underbrace{(-15x+144)}_{\text{sandwiches sold}}\underbrace{(x-2)}_{\text{profit per sandwich}} = -15x^2+174x-288$$



$h = \frac{-174}{2(-15)} = \boxed{\$5.80}$

6. Let $f(x)$ be a linear-to-linear rational function whose graph passes through the points $(0,-4)$, $(3,-16)$, and $(4,12)$.

(a) **[5 points]** Write a formula for $f(x)$.

$$f(x)=\frac{ax+b}{x+d}$$

$$-4=\frac{b}{d} \longrightarrow b=-4d$$

$$-16=\frac{3a+b}{3+d} \longrightarrow 3a+b=-48-16d$$

$$12=\frac{4a+b}{4+d} \longrightarrow 4a+b=48+12d$$

$$3a=-48-12d \longrightarrow a=-16-4d$$

$$a=96+28d \longrightarrow 96+28d=-16-4d$$

$$112=-32d$$

$$\boxed{d=-3.5}$$

$$a=-16-4(-3.5)$$

$$\boxed{a=-2}$$

$$b=-4d$$

$$\boxed{b=14}$$

$$\boxed{f(x)=\frac{-2x+14}{x-3.5}}$$

(b) **[2 points]** Find the asymptotes of $f(x)$.

$$\boxed{\begin{array}{l} y=-2 \\ x=3.5 \end{array}}$$

(c) **[3 points]** Let $g(x)=f(f(x))$. Find the asymptotes of $g(x)$.

$$g(x)=f(f(x))=\frac{-2\left(\frac{-2x+14}{x-3.5}\right)+14}{\left(\frac{-2x+14}{x-3.5}\right)-3.5}\cdot\frac{(x-3.5)}{(x-3.5)}$$

$$=\frac{-2(-2x+14)+14(x-3.5)}{-2x+14-3.5(x-3.5)}=\frac{28x-77}{-5.5x+26.25}=\frac{\frac{-56}{11}x+14}{x-\frac{105}{22}}$$

Asymptes: $\boxed{y=\frac{-56}{11} \qquad x=\frac{105}{22}}$

7. **[10 points]** Eleanor and Jason are walking around the coordinate plane.

Eleanor starts at the origin. She walks west at a speed of 80 meters per minute for 1 hour.

Jason also starts at the origin, but his path is a little more complicated:

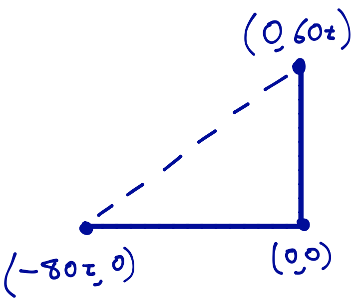
First, he walks north at a speed of 60 meters per minute for 30 minutes.

Then, he waits in place for 10 minutes.

Finally, he walks to the west at a speed of 90 meters per minute for 20 minutes.

Write a multipart function for the distance between Eleanor and Jason after t minutes.

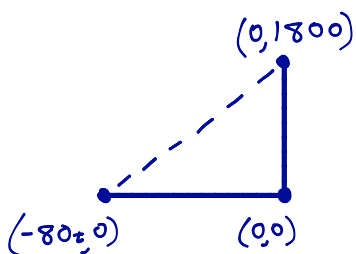
For $0 \le t \le 30$:



$$d = \sqrt{(80t)^2 + (60t)^2}$$

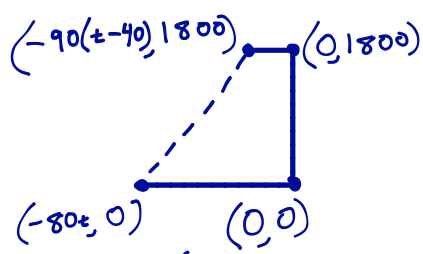
$$= 100t$$

For $30 \le t \le 40$:



$$d = \sqrt{(80t)^2 + 1800^2}$$

For $40 \le t \le 60$:



$$d = \sqrt{(-80t + 90(t-40))^2 + 1800^2}$$

$$= \sqrt{(10t - 3600)^2 + 1800^2}$$

$$\boxed{d(t) = \begin{cases} 100t & \text{if } 0 \le t \le 30 \\ \sqrt{(80t)^2 + 1800^2} & \text{if } 30 \le t \le 40 \\ \sqrt{(10t - 3600)^2 + 1800^2} & \text{if } 40 \le t \le 60 \end{cases}}$$