Math 125 **Final Examination** **Spring 2014**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 total points) Evaluate the following indefinite integrals.

   (a) (5 points) $\displaystyle\int \frac{x}{\sqrt{x+2}}\,dx$

   (b) (5 points) $\displaystyle\int e^{2x}\sec(e^{2x})\tan^3(e^{2x})\,dx$

2. (10 total points) Evaluate the following definite integrals.

   (a) (5 points) $\displaystyle\int_1^2 \frac{\ln x}{x^3}\,dx$

   (b) (5 points) $\displaystyle\int_2^3 \sqrt{4x-x^2}\,dx$

3. (10 points) Find the area under the curve

$$y = \frac{1}{\sqrt{|2x - x^2|}}$$

and above the x-axis, for $-1 \leq x \leq 1$. *CAUTION*: The integral you will get is an improper integral. Be sure to treat your integral as an improper integral, and justify your answer.

4. (10 points) Let $A(t)$ denote the area under the curve $y = \sqrt{1-x^3}$ and above the x-axis, between the vertical lines $x = t$ and $x = 2t$.
   Find the value of t for which $A(t)$ is a maximum on the interval $0 \le t \le 1/2$.
   Justify that your answer gives the maximum.

5. (10 points) The region under the curve $y = 2x - x^2$ and above the x-axis is rotated around the line $x = -b$, where b is a positive constant. Find the value of b for which the volume of the solid so obtained is 10π.

6. (10 points) An 80-ft cable is used to lift 50 pounds of coal up a mine shaft 80 ft deep. The bottom half of the cable weighs 2 pounds per foot and the top half of the cable weighs 3 pounds per foot. Find the work done in foot-pounds.

7. (10 total points) This problem gives one way to find a rational number that approximates π.

   (a) (4 points) Show that $\displaystyle\int_0^4 \frac{dy}{1+(y^2/16)} = \pi.$

   (b) (6 points) Subdivide $[0,4]$ into 4 equal subintervals and use Simpson's rule to approximate the integral in part (a). You do *NOT* have to simplify any expressions involving fractions.
   (For example, if you have terms that look something like $\dfrac{2}{1+\frac{81}{16}}$, just leave them in that form.)

8. (10 points) Let $\mathscr{R}$ be the region above the x-axis, below the graph of $y = \dfrac{1}{(x+1)(x+2)}$, between $x = 1$ and $x = 2$. Find the x-coordinate of the centroid (center of mass) of the region $\mathscr{R}$.

9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dt} = \frac{1}{ye^{t/2}}, \quad y(0) = -1.$$

Give your answer in the form $y = f(t)$.

10. (10 points) A tank initially contains 15 liters of pure water. Sea water with a salt concentration of 35 grams per liter is added at a rate of 2 liters per minute. In addition, pure water is added at a rate of 1 liter per minute. The solution is kept thoroughly mixed and is drained from the tank at a rate of 3 liters per minute. How much salt is in the tank after t minutes?