Math 125 **Final Examination** **Spring 2015**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 total points) Evaluate the following indefinite integrals.

   (a) (5 points) $\int t^5 \sin(t^3)\, dt$

   (b) (5 points) $\int \frac{1}{x(x+\sqrt{x})}\, dx$

2. (10 total points) Evaluate the following definite integrals.

   (a) (5 points) $\displaystyle\int_0^1 \ln(1+t^2)\,dt$

   (b) (5 points) $\displaystyle\int_0^2 \frac{x^3}{\sqrt{4+x^2}}\,dx$

3. (10 points) Determine whether the following improper integral is convergent or divergent. If it is convergent, evaluate it.

$$\int_0^1 \frac{1}{\sqrt{x(1-x)}}\,dx.$$

4. (10 points) A particle is moving along a straight line. For time $t \geq 0$, the velocity of the particle is given by

$$v(t) = 3t^2 - 12.$$

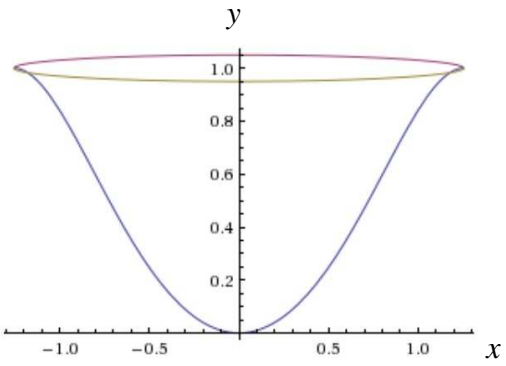
Let b be an arbitrary number greater than 10. Find the *total distance* traveled by the particle from time $t = 0$ to time $t = b$. Your answer should be an expression involving b.

5. (10 total points) The triangle whose vertices have (x,y) coordinates (0,1), (1,0) and (2,0) is rotated around the vertical line $x = -1$ to form a solid of revolution.

   (a) (5 points) Using *WASHERS*, set up a definite integral (or the sum of two definite integrals if necessary) for the volume of this solid of revolution. DO NOT EVALUATE THE INTEGRAL(S).

   (b) (5 points) Using *SHELLS*, set up a definite integral (or the sum of two definite integrals if necessary) for the volume of this solid of revolution. DO NOT EVALUATE THE INTEGRAL(S).

6. (10 total points) The curve $x = \sqrt{\sin^{-1} y}$ for $0 \le y \le 1$ is rotated around the y-axis to form a container. The container is filled with a fluid that weighs 40 lb/ft^3. Length units for x and y are in feet.



(a) (6 points) Set up a definite integral (with respect to y) for the work (in ft-lb) required to empty the container by pumping all of the fluid to the top of the container.

(Note: Do not use the acceleration due to gravity; pounds are already a unit of force.)

IN THIS PART, DO NOT EVALUATE THE INTEGRAL YET.

(b) (4 points) Now evaluate the integral in part (a). Give your answer in exact form.

7. (10 total points)

   (a) (5 points) Set up a definite integral for the arclength of the curve $y = \frac{1}{3}x^3 + \frac{1}{2}x^2$ for $-2 \leq x \leq 2$. DO NOT EVALUATE THE INTEGRAL.

   (b) (5 points) Use Simpson's rule with $n = 4$ subintervals to approximate the definite integral in part (a). Give your answer in exact form.

8. (10 points) Find the x-coordinate $\bar{x}$ of the centroid (center of mass) of the region in the first quadrant inside the circle of radius r centered at the origin. (The region is a quarter of a circle.)
Give your answer in exact form. Your answer will involve r.

9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dx} = 2xe^{-tan(y)}\cos^2(y), \quad y(0) = \frac{\pi}{4}.$$

Give your answer in the form $y = f(x)$.

10. (10 total points) A person borrows \$20,000 and repays the loan at the rate of A dollars per year. The lender charges interest on the loan at a rate of 5% per year. Assuming that payments are made continuously and that interest is compouned continuously, the amount $y(t)$ of money (in dollars) owed t years after the loan is made satisfies the differential equation

$$\frac{dy}{dt} = \frac{1}{20}y - A$$

and the initial condition

$$y(0) = 20,000.$$

(a) (6 points) Solve this intitial-value problem (the differential equation together with the initial condition) to get a formula for $y(t)$. Your answer will involve both t and A.

(b) (4 points) Find the value of A which makes $y(10) = 0$ (so the loan is paid off in 10 years). Give your answer in exact form.