**Math 125** **Final Examination** **Spring 2016**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can use only Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.

| Question | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |

| Question | Points | Score |
|---|---|---|
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| Total | 90 | |

1. (10 total points) Evaluate the following indefinite integrals.

   (a) (5 points) $\displaystyle\int \frac{\ln(x)}{x^2}\,dx$

   (b) (5 points) $\displaystyle\int \tan^3 t\,dt$

2. (10 total points) Evaluate the following definite integrals.

   (a) (5 points) $\displaystyle\int_1^4 \frac{1}{\sqrt{y}\left(2y-\sqrt{y}\right)}\,dy$

   (b) (5 points) $\displaystyle\int_{-1}^{-2+\sqrt{2}} \frac{\sqrt{x^2+4x+3}}{x+2}\,dx$

3. (10 total points) The velocity of a particle moving along the number line is $v(t) = 4t(t^2 - 1)^{1/3}$ ft/sec. The particle starts at position $s(0) = 5$ feet.

   (a) (5 points) Find the function, $s(t)$, for the position of the particle at time $t$ seconds.

   (b) (5 points) Find the **total distance** traveled by the particle from $t = 0$ to $t = 3$ seconds.

4. (10 points) Consider the improper integral

$$\int_1^5 \frac{dx}{x^2\sqrt{25-x^2}}.$$

Evaluate this integral or explain why it does not converge.

5. (10 points) Let $\mathscr{A}$ be the region in the first quadrant bounded by $y = x^2$, $y = 25$, and the $y$-axis. Find the value of $m$ with the property that the line $y = mx$ divides $\mathscr{A}$ into two regions with the same area.

6. (10 points) Let $\mathscr{R}$ denote the region in the $xy$-plane enclosed by $y = \sin(x)$ and the $x$-axis, between $x = 0$ and $x = \pi$. Which solid of revolution has a larger volume: the one obtained by rotating this region $R$ around the $x$-axis, or the one obtained by rotating $\mathscr{R}$ around the y-axis?

   Justify your answer by computing **both** volumes. No credit for guessing.

7. (10 points) A cable that weighs 4 lbs/ft is used to lift a sandbag up a mine shaft 50 feet deep. A small tear in the bag causes sand to leak out at a constant rate as the sandbag is lifted. The sandbag weighs 80 pounds initially and 60 pounds when it gets to the top.

   How much work is done in total to lift the sandbag **and** the cable to the top?

   (Hint: Find the weight of the sandbag as a *linear* function of its height above the ground.)

8. (10 points) Solve the initial value problem. Find an explicit formula for $y$ as a function of $x$.

$$y' = x^2 + x^2 y^2, \qquad y(0) = 1.$$

9. (10 points) At 10:07 P.M. you find a secret agent murdered. Next to her is a martini that got shaken before the secret agent could stir it. The room temperature is 70°F. The martini warms from 60°F to 61°F in the 2 minutes from 10:07 P.M. to 10:09 P.M. If the secret agent's martinis are always served at 40°F, what was her time of death, to the nearest minute?

   Recall that Newton's Law of Cooling states that the rate of change of the temperature of the martini is proportional to the temperature difference between the martini and its surroundings.