Math 125

Final Examination

Your Name

Student ID #

Your	Signature	
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TA's Name

Quiz Section

Professor's Name

• Turn off all cell phones, pagers, music players, and other similar devices.

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must show all of your work. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.

• Place a box around your answer to each question. Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.

- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	10	
4	12	
5	12	

Question	Points	Score
6	10	
7	12	
8	10	
9	10	
Total	100	

1. (12 points) Evaluate the following integrals. $f = \frac{11}{2}r - \frac{12}{2}$

(a) (6 points)
$$\int \frac{11x - 12}{x^3 - 4x^2 + 4x} dx$$

(b) (6 points)
$$\int \frac{\tan^2(t) \sec^2(t)}{1 + \tan(t)} dt$$

- 2. (12 points)
 - (a) (8 points) Evaluate the indefinite integral: $\int \frac{1}{(x^2 + 2x + 2)^{3/2}} dx$

(b) (4 points) Use your answer from part (a) to determine if the following improper integral is convergent or divergent. If convergent, evaluate it. Show all limit computations.

$$\int_0^\infty \frac{1}{(x^2 + 2x + 2)^{3/2}} \, dx$$

3. (10 points) Consider the region between the curve y = f(x), the line x = 1, and the line y = 4. A formula for f(x) is not known; however, we do have the following picture of the region and a table of values:



(a) (6 points) Use Simpson's Rule with n = 6 subintervals to estimate the volume of the solid of revolution obtained by rotating this region around the **horizontal** line y = 4.

(b) (4 points) Define $A(x) = \int_{1}^{1+1/x} f(t) dt$, where f is the same function as in the table above. Compute A'(2). 4. (12 points) The velocity of a particle moving along the number line is given at all times $t \ge 1$ by

$$v(t) = \frac{8}{t^3} - \frac{4}{t^2}$$
 ft/sec.

At time t = 2 seconds, the particle's position is s(2) = 10 feet.

(a) (6 points) Find the function, s(t), for the position of the particle at time t seconds, $t \ge 1$.

(b) (6 points) Find the **total distance** traveled by the particle from t = 1 to t = 4 seconds.

5. (12 points) Find the volume of the body of revolution obtained by rotating the region below $y = \frac{e^{\sqrt{x}}}{\sqrt{x}}$ and above the *x*-axis, between x = 4 and x = 9, about the **vertical** line x = -1.



6. (10 points) A water tower has a tank that has a hemispherical shape. Its dimensions are shown in the figure. The bottom of the tank is 40 meters above the ground. Water is pumped into the tower from a well 10 meters under ground.

Compute the work required to completely fill the empty tank with water.

Assume that water weighs 1000 kg/m³, the gravitational acceleration is g = 9.8 m/s², and that the amount of water in the pipe between the water table and the tank is negligibly small.



7. (12 points) Find the center of mass of the region under the graph of $y = \sin(x)$, above the *x*-axis, between x = 0 and $x = 2\pi/3$ radians.

You may give your answer in exact, simplified form, or in decimal form, rounded to the nearest 4 decimal digits.

8. (10 points) Find the explicit solution y = y(x) for the differential equation:

$$\frac{dy}{dx} = \left(\tan(x)\sec^2(x) - 4\right)e^{2y}$$

subject to the initial condition:

$$y(0) = 1.$$

9. (10 points) At time t = 0, a tank contains 100 gallons of pure gasoline. A mixture whose volume is 30% ethanol and 70% gasoline is pumped into the tank at a rate of 2 gallons per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate.

How many gallons of ethanol will there be in the tank after 50 minutes? Give your answer as a decimal number, rounded to the nearest 4 decimal digits.