Math 125 **Final Exam** **Spring 2018**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas # 1-18 in the table from section 7.5 of your textbook (posted on the departmental math 125 website), without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example: $\frac{\pi}{3} + e^2 - 5\sqrt{3}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Evaluate the following integrals. Show your work. Simplify and box your answers.

   (a) (5 points) $\displaystyle\int \frac{1}{x^4 - x^2}\,dx$

   (b) (5 points) $\displaystyle\int \sin^3(t)\,dt$

2. Evaluate the following integrals. Show your work. Simplify and box your answers.

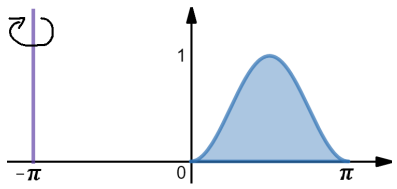
   (a) (5 points) $\displaystyle\int_{-1}^{0} \frac{x}{x^2+2x+2}\,dx$

   (b) (5 points) $\displaystyle\int e^{2x}\sin x\,dx$

3. (10 points) Find the arc length of the curve $y = \frac{1}{16}e^{2x} + e^{-2x}$ for $0 \leq x \leq 1$.

4. (10 points) The **position** of a particle at time t is given by $s(t) = t^3 - 2t^2 - 4t + 20$.
   Find the total distance traveled by the particle from $t = 0$ to $t = 3$.

5. The region below $y = \sin^2 x$, above the x-axis, between $x = 0$ and $x = \pi$ is rotated about the line $x = -\pi$ to generate a solid of revolution.

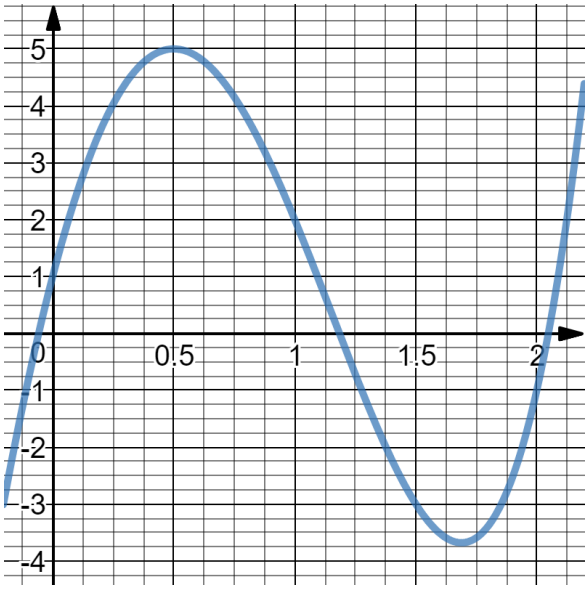


(a) (5 points) Write down an integral equal to the volume of the resulting solid of revolution. Do not compute the integral yet.

(b) (5 points) Compute the integral to determine the volume of the solid of revolution. Give your answer in exact form or as a decimal number with at least four significant digits.

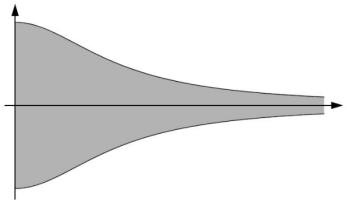
6. (10 points) The graph of a function $f(t)$ is shown. Use it to answer the following questions.

(a) (5 points) Approximate the **average value** of this function over the interval $[0,2]$ using Simpson's Rule with $n = 4$ subintervals.



(b) (5 points) Define a new function: $g(x) = \int_x^{2x} f(t)dt$, where f is the function in the graph above. Compute $g'(0.5)$

7. Consider the region bounded above by the graph of $y = \dfrac{1}{1+x^2}$, bounded below by the graph of $y = \dfrac{-1}{1+x^2}$, bounded on the left by the y axis, and with no bound on the right.



(a) (5 points) Find the area of this region. If it's finite, compute its value. If it's infinite, show why. Show all steps and limit computations in either case.

(b) (5 points) Find the $\bar{x}$ coordinate of the center of mass of this region. If it's finite, compute its value. If it's infinite, show why. Show all steps and limit computations in either case.

8. (10 points) A part of an elevated freeway is 30 meters long and has half elliptical cross sections, as shown below. The width at the top of the cross section (the horizontal axis of the ellipse) is 4 meters. The height in the middle of the cross section (half of the vertical axis of the ellipse) is 1 meter. The lowest point of the section is 5 meters above the ground.

   The structure is made of concrete which was pumped from the ground level up, in liquid state. The mass density of concrete is 2,400 kg per cubic meter. The gravitational acceleration is 9.8 m/sec^2.

   SET UP (do NOT evaluate) an integral equal to the WORK that was done to pump the concrete up to make the section of the structure. The work done to make the support column should be ignored.

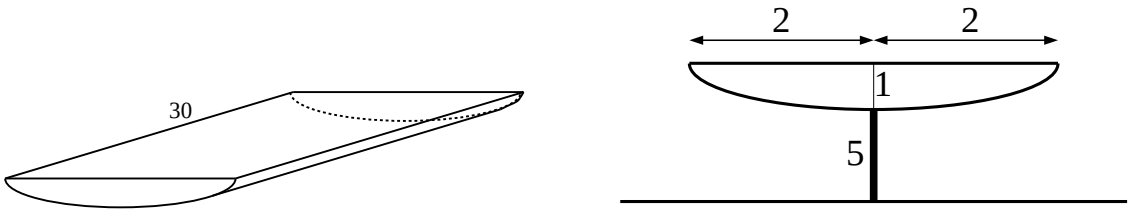


Figure 1: *The upper part of the structure is depicted on the left. The cross section is depicted on the right. Recall that the equation of an ellipse* ***centered at the origin*** *is* $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$*, where a and b are half of the horizontal and vertical axes of the ellipse, respectively.*

9. (10 points) Find the solution of the initial value problem

$$y' = (y^2+4)^{3/2}\, x \ln x \quad \text{with } y(1) = 2.$$

Leave your answer as an implicit equation involving the variables y and x.

10. At this time, the population of Mountainstan is 100 million people. The natural growth rate (births minus deaths) is 2% of population per year. Each year 1 million people emigrate from Mountainstan to other countries looking for better economic opportunities. Assume no people move from other countries to Mountainstan.

    (a) (4 points) Let $y = y(t)$ be the population of Mountainstan, in millions, t years from now. Write down a differential equation for y. Do NOT solve the equation yet.

    (b) (6 points) Assuming that these trends will be stable for a long time, determine how many years from now the population will reach 150 million. Give your answer as a decimal number with at least 4 significant digits.