## Math 125

## **Final Examination**

Your Name	Your Signature	
Student ID #	Quiz Section	
Professor's Name	TA's Name	

- This exam is closed book. You may use one  $8.5'' \times 11''$  sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example  $\frac{\pi}{3}$  or  $5\sqrt{3}$ ), except as noted in particular problems.
- A scientific calculator is needed, but no calculator with graphing, programming, symbolic manipulation, or calculus capabilities is allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

- 1. (10 total points) Evaluate the following definite integrals. Give your answers in exact form.
  - (a) (5 points)  $\int_0^1 (x-3) e^{-2x} dx$

(b) (5 points) 
$$\int_0^2 \frac{1}{(x^2+4)^{3/2}} dx$$

2. (10 total points) Evaluate the following indefinite integrals.

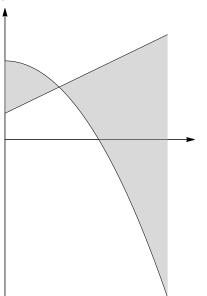
(a) (5 points) 
$$\int (\sin x + \cos^2 x) \sin x \, dx$$

(b) (5 points) 
$$\int \frac{1}{(x+1)(\sqrt{x+5}+1)} dx$$

3. (10 points) Let *R* be the region that is between the curve  $y = \sqrt{x}e^{-x^2}$  and the *x*-axis, is bounded on the left by the line x = 1, and extends infinitely far out to the right. Let *S* be the solid obtained by rotating *R* around the *x*-axis.

Does *S* have finite volume? If so, find it, and give your answer in exact form.

4. (10 points) Find the total area (shaded in the figure below) that is between the curves  $y = 3 - x^2$  and y = x + 1 and between the lines x = 0 and x = 3.

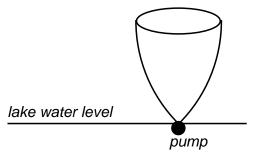


5. (10 points) Let *T* be the triangle with vertices (0,1), (4,9) and (5,6). Using the method of *shells* (*not* washers), write the volume of the solid obtained by rotating *T* about the line y = -1 in terms of definite integrals. DO NOT EVALUATE THE INTEGRAL(S).

6. (10 points) A water container has the shape of a solid of revolution, obtained by rotating the region below the line  $y = \pi/2$ , above the curve  $y = \sin^{-1} x$ , and between the lines x = 0 and x = 1, about the *y*-axis. Dimensions on the *x*- and *y*-axes are in meters.

The container is placed above the surface of a lake, so that its lower end touches the water (see the figure below). The lower end contains a small opening through which a small but powerful pump can fill the container with water. Calculate the work needed to fill the empty container with water pumped from the lake up into the container. Give your answer in exact form.

Assume that the gravitational acceleration is 9.8 meters per second squared. The density of water is 1,000 kg per cubic meter.



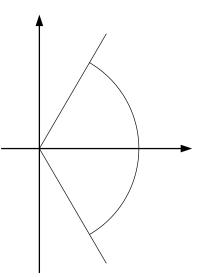
- 7. (10 total points)
  - (a) (4 points) Write a definite integral for the arclength *L* of the graph of  $y = \sin(2x)$  from x = 0 to  $x = \pi/3$ .

(b) (6 points) Use the Trapezoid Rule with n = 4 subintervals to approximate the definite integral in part (a). Give your answer first in exact form, and then give a decimal approximation with at least five digits after the decimal point.

## 8. (10 points)

Find the *x*- and *y*-coordinates of the center of mass of a uniform flat plate that is below the line  $y = \sqrt{3}x$ , above the line  $y = -\sqrt{3}x$ , and bounded on the right by the circle of radius 2 centered at the origin (see the figure).

Give your answer in exact form.



9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{(x+3)(y+2)}{x^2+9}, \quad y(0) = 10.$$

Give your answer in the form y = f(x).

10. (10 points) A large vat initially contains 10 liters of pure water. A solution of salt in water is pumped into the vat at a rate of 2 liters per hour. The incoming solution contains b grams of salt per liter. The solution in the vat is kept thoroughly mixed and is drained from the vat at a rate of 2 liters per hour. After 8 hours, the concentration of salt in the vat is 3 grams per liter.

What is the concentration b of salt in the incoming mixture?