

Your Name

Your Signature

Student ID #

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Quiz Section

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Professor's Name

TA's Name

- Turn off all cell phones, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can use only a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet.**
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	10	
4	12	
5	12	

Question	Points	Score
6	10	
7	10	
8	12	
9	10	
Total	100	

1. (12 points) Evaluate the following integrals. Box your final answer.

(a) (6 points) $\int \frac{\sin^2(x) \tan(x)}{\sec(x)} dx$

(b) (6 points) $\int \frac{x^3 + 2x^2 + 4x + 6}{x^3 + 3x} dx$

2. (12 points)

Evaluate the following definite integrals. Simplify your answer, but leave it in exact form.

(a) (6 points) $\int_1^4 \sqrt{\sqrt{x} - 1} \, dx$

(b) (6 points) $\int_{-2}^2 \frac{x^2}{\sqrt{16 - x^2}} \, dx$

3. (10 points) Evaluate the following improper integral showing all the appropriate steps. If the integral diverges, then say so.

$$\int_0^{\infty} xe^{-2x} dx$$

4. (12 points) Consider the region, R , bounded by the curve $y = x^3$, the **vertical** line $x = 2$, and the x -axis.
- (a) (6 points) Find the value of the constant a such that the **vertical** line $x = a$ divides the region R into two regions of equal area.

- (b) (6 points) A solid is obtained by rotating the region R around the **horizontal** line $y = -1$. Set up the integrals you get for the volume of this solid using **BOTH** the method of cylindrical shells and the method of washers (DO NOT EVALUATE).

Shells:

Washers:

5. (12 points) A cable that weighs 2 lbs per foot is used to lift a bucket of water from a well. The bucket of water weighs 20 lbs, and it needs to be lifted 10 feet to reach the top of the well.

How far is the bucket from **the bottom of the well** when only half of the total work was done?

6. (10 points) Solve the differential equation:

$$\frac{dy}{dx} = 2 \cos^2(x) \cos^2(y) - \cos^2(y)$$

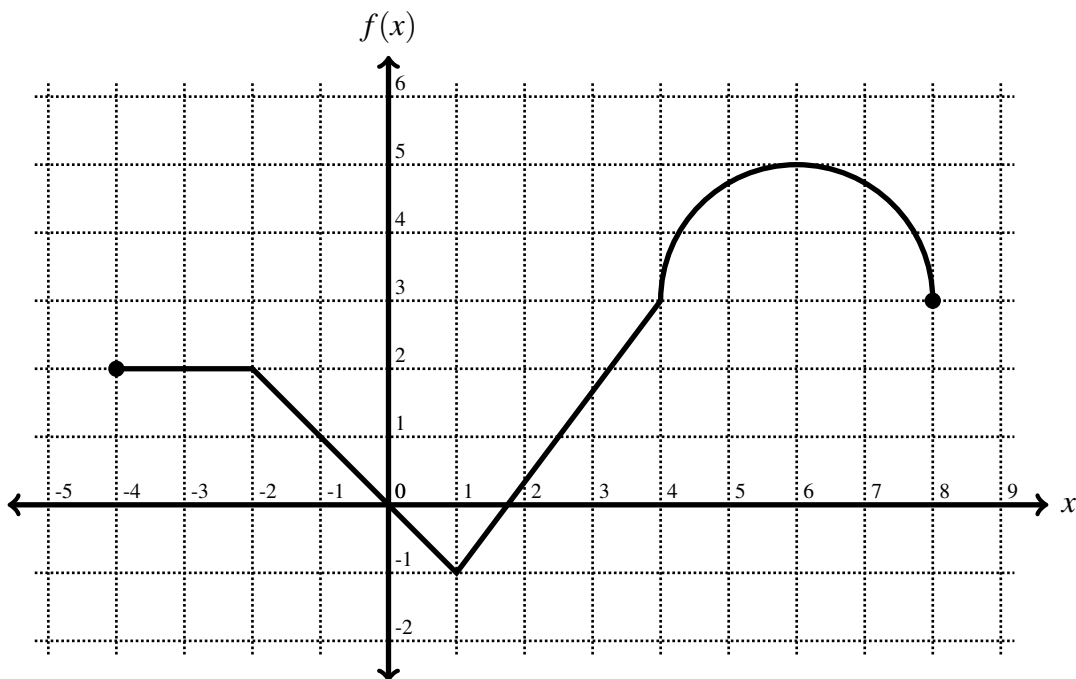
subject to the initial condition

$$y(\pi/4) = \pi/6$$

Give your answer in the form $y = f(x)$.

7. (10 points) A particle is moving along a straight line. Its acceleration at t seconds is $a(t) = t \text{ m/s}^2$. The particle begins at the origin, and returns to the origin after 6 seconds. What is the **total distance** traveled by the particle during that time?

8. (12 points) The graph of $f(x)$ is shown below. It consists of line segments and a half circle. Use it to answer the following questions.



- (a) Compute the average value of $f(x)$ on the interval $[-4, 1]$.

(b) Compute $\int_4^6 \sqrt{1 + [f'(x)]^2} dx$

(Hint: Think first. You don't need more space than you have here to answer this question!)

(c) Compute $\int_4^6 x f'(x) dx$

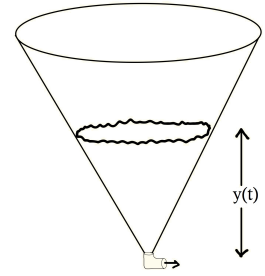
9. (10 points)

Water is flowing out of a conical container. Let $y(t)$ denote the depth (in feet) of the water in the container and let $V(t)$ denote the volume (in cubic feet) of the water at time t (in minutes). It is known that:

$$\frac{dV}{dt} = -6\sqrt{y}$$

Moreover, V and y are related by the equation:

$$V = 5y^3$$



- (a) (2 points) Differentiate the second equation **with respect to time t** .
- (b) (2 points) Use your answer in part (a) to get a differential equation for y .
- (c) (6 points) If the initial depth of the water in the container is 4 feet, how long does it take to empty the container?