

Your Name

Your Signature

Student ID #

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Quiz Section

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Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below. You must **show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	2. $\int \frac{1}{x} dx = \ln x $
3. $\int e^x dx = e^x$	4. $\int b^x dx = \frac{b^x}{\ln b}$
5. $\int \sin x dx = -\cos x$	6. $\int \cos x dx = \sin x$
7. $\int \sec^2 x dx = \tan x$	8. $\int \csc^2 x dx = -\cot x$
9. $\int \sec x \tan x dx = \sec x$	10. $\int \csc x \cot x dx = -\csc x$
11. $\int \sec x dx = \ln \sec x + \tan x $	12. $\int \csc x dx = \ln \csc x - \cot x $
13. $\int \tan x dx = \ln \sec x $	14. $\int \cot x dx = \ln \sin x $
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$	18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place a box around your answer to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All pages are double-sided except for this cover page and the last page. You may use the blank pages for extra room but, if you want us to grade these spare pages, clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank page(s) at the end of the exam.
- This exam has 10 problems on 10 pages. When the exam starts, check that your exam is complete. Good luck!

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1. (10 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

(a) $\int x \tan^2 x dx$

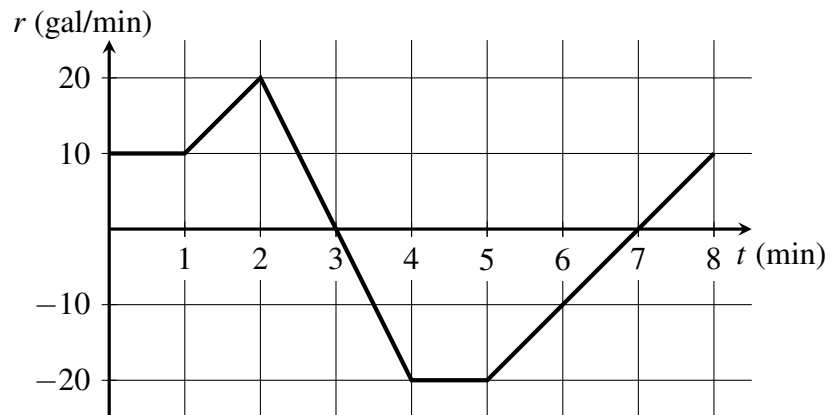
(b) $\int_{-2}^4 |x+1| dx$

2. (10 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

(a) $\int \frac{1}{1+x^{2/3}} dx$

(b) $\int x\sqrt{5+4x-x^2} dx$

3. (10 points) The following graph depicts a function $r(t)$ that gives the rate of flow of water into a tank, measured in gallons per minute. At time $t = 0$ minutes, the tank contains 100 gallons of water.



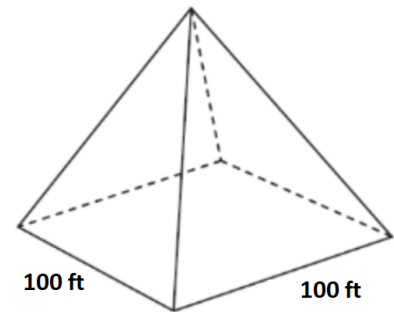
- (a) What is the maximum amount of water in the tank at any time? Show work/justify.
- (b) At which time(s) is the amount of water in the tank decreasing the fastest?
- (c) At which time(s) does the tank have the least amount of water in it?
- (d) What is the average rate of flow of water into the tank over the first 4 minutes? Show work.

(e) Let $f(x) = \int_1^{x^2} r(t) dt$. Find $f'(2)$, showing your steps.

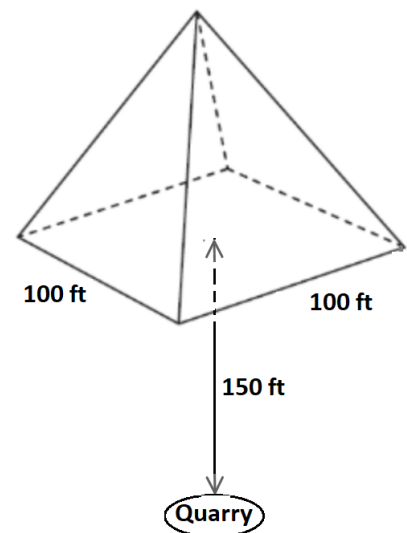
4. (10 points) A high-speed bullet train travels between two consecutive stations that are 30 km apart. Find the time it takes the train to travel between these two stations, if the train starts at rest at the first station, accelerates at 10 km/min^2 until it reaches its maximum cruising speed of 3 km/min , drives at that speed for as long as possible, then decelerates at 5 km/min^2 in time to stop at the second station.

5. (10 points) A solid pyramid that is 60 feet tall with a square base that is 100 feet wide is to be built out of limestone. (A cubic foot of limestone weighs 175 pounds.)

- (a) Set up (but DO NOT EVALUATE) an integral for the **total weight, in pounds**, of the limestone needed to build the pyramid.



- (b) Set up (but DO NOT EVALUATE) an integral for the **work, in foot-pounds**, needed to raise the limestone to the required height to build the pyramid, assuming all limestone starts in a quarry that is 150 feet below the base of the pyramid.



6. (10 points) The region in the first quadrant to the right of the y -axis, above the curve $y = 5x^2$, and below the curve $y = x^2 + 1$ is rotated around the y -axis to form a solid of revolution.
- (a) Set up a definite integral for the volume of this solid using the method of cylindrical shells, and evaluate the integral to compute the volume.

- (b) Using the washers/disks method, write the volume of this solid in terms of definite integrals. DO NOT EVALUATE the integrals.

7. (a) (4 points) Set up a definite integral for the arclength of the curve

$$y = \sin(2x) \text{ for } 0 \leq x \leq \pi/4.$$

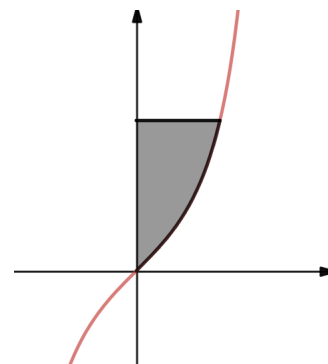
DO NOT EVALUATE THIS INTEGRAL.

- (b) (6 points) Approximate the integral in part (a) using the Trapezoid Rule with $n = 3$ subintervals. Simplify your answer, but leave it in exact form, as an expression in terms of π and square roots, evaluating all trig functions.

8. (10 points) The region shown below is bounded by the curves

$$y = \sec(x) \tan(x) \text{ and } y = \sqrt{2}, \text{ for } 0 \leq x \leq \pi/4.$$

Compute the y-coordinate, \bar{y} , for the center of mass of a thin lamina of uniform density occupying this region. *Show all work and box your answer. Give your answer in decimal form, rounded to 2 decimal digits.*



9. (10 points) Solve the following differential equation, subject to the stated initial value. Show all steps, and give your answer in explicit form, $y = f(x)$.

$$(x^2 - 2x)y' = (x - 4)y, \quad y(1) = -3$$

10. A large vat contains 10 liters of brine (salt dissolved in water). More brine is pumped into the vat at a rate of 2 liters per hour. The incoming brine solution contains 3 grams of salt per liter. The solution in the vat is kept thoroughly mixed and is drained from the vat at a rate of 2 liters per hour.
- (a) (3 points) Set up a **differential equation** for the amount $y = y(t)$ of grams of salt in the vat at t hours. Do not solve yet.
- (b) (5 points) Denote by y_0 the initial amount of salt in the vat, in grams, at time $t = 0$ hours. Solve the differential equation to find $y(t)$. Your answer will include the unknown constant y_0 .
- (c) (2 points) Suppose that after 4 hours, the concentration of the salt in the vat is 4 grams per liter. What was the initial concentration of the salt in the vat (in grams per liter)?

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