MIDTERM 2 ANSWERS
MATH 125

Problem 1. *(13 points total)* Compute the following integrals. Give your answer in exact form (do not use decimals).

a. *(6 points)* $\displaystyle\int \frac{2x-3}{x^2+2x+1}\,dx$

Solution: partial fractions. Get $\displaystyle 2\ln|x+1| + 5\frac{1}{x+1} + C.$

b. *(7 points)* $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\ln(\tan\theta)}{\sin\theta\,\cos\theta}\,d\theta$

Solution: substitution! Guess: $u = \ln(\tan\theta)$. *Get* $\displaystyle\frac{(\ln\sqrt{3})^2}{2}.$

Problem 2. *(8 points total) Compute the following integral. Give your answer in exact form (do not use decimals).*

$$\int \frac{t^3}{\sqrt{t^2+4}}\,dt$$

Solution: trig substitution. Get $\displaystyle\frac{(t^2+4)^{3/2}}{3} - 4\sqrt{t^2+4} + C.$

Problem 3. *(9 points total) Find a postitive number h such that the average value of the function*

$$f(x) = x^2 - 5 - 2x$$

on the interval $[0, h]$ *equals* 1.

Solution: $h = 6$.

Problem 4. *(9 points) Use Simpson's Rule with $n = 4$ subintervals to approximate the integral*

$$\int_3^5 \frac{e^x}{x}\,dx.$$

Solution: approximately 30.25

Problem 5. *(11 points)* Determine whether each of the following integrals is convergent or divergent. If it is convergent, evaluate it.

a. *(5 points)* $\displaystyle\int_1^\infty \frac{2x+3}{x^2+3x+1}\,dx.$

Solution: use substitution. Integral is divergent.

b. *(6 points)* $\displaystyle\int_0^\infty xe^{-x}\,dx.$

Solution: use integration by parts and L'Hospital's rule. Integral is convergent and equal to 1.