Integration by Parts | Math 125

Name ______________________ Quiz Section ______

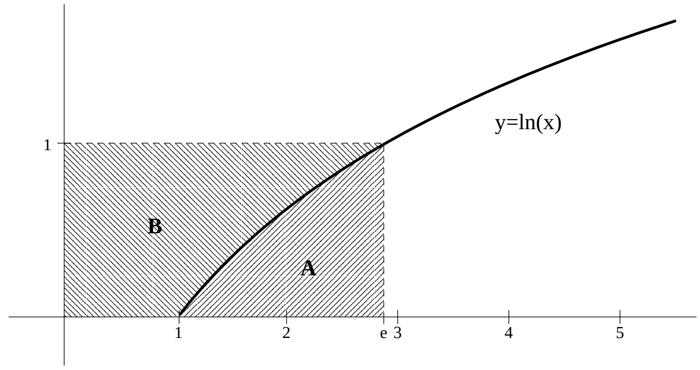
In this work sheet we'll study the technique of integration by parts. Recall that the basic formula looks like this:

$$\int u\,dv = u \cdot v - \int v\,du$$

1 First a warm-up problem. Consider the integral $\int x\,\sin(3x)\,dx$. Let $u = x$ and let $dv = \sin(3x)\,dx$. Compute du by differentiating and v by integrating, and use the basic formula to compute the original integral. Don't forget the arbitrary constant!

2 Compute $\int \ln x\,dx$. (The proper technique is, indeed, integration by parts. What should you take to be u and dv? The choices are pretty limited. Try one and see what happens.)

3 The regions A and B in the figure are revolved around the x-axis to form two solids of revolution.



(a) Before computing the integrals, which solid do you think has a larger volume? Why?

(b) Use the disk method to find the volume of the solid swept out by region A.

(c) Use the shell method to find the volume of the solid swept out by region B.

4 Suppose we try to integrate $1/x$ by parts, taking $u = 1/x$ and $dv = dx$. We have $du = (-1/x^2)\,dx$ and $v = x$, so

$$\begin{aligned} \int \frac{1}{x}\,dx &= \frac{1}{x}\cdot x - \int x \cdot \frac{-1}{x^2}\,dx \\ &= 1 + \int \frac{1}{x}\,dx. \end{aligned}$$

Canceling the integral from both sides, we get the disconcerting result that $0 = 1$. *What went wrong?* What happens if we replace the indefinite integrals by definite integrals, that is, if we try to calculate $\int_a^b \frac{1}{x}\,dx$ by this method?