Homework #1 Math 126

These problems are based on the material in Section 1 of the Taylor Notes.

1. Find the first Taylor polynomial $T_1(x)$ for $f(x)$ based at b and use the Tangent Line Error Bound to bound the error $|f(x) - T_1(x)|$ on the interval I where
   (a) $f(x) = e^x \quad b = 0 \quad I = [-1, 1]$.
   (b) $f(x) = \ln(1 + x) \quad b = 0 \quad I = [-\frac{1}{2}, \frac{1}{2}]$.
   (c) $f(x) = \sin(x) \quad b = 0 \quad I = [-0.1, 0.1]$.
   (d) $f(x) = x^{\frac{1}{3}} \quad b = -8 \quad I = [-9, -7]$.

2. For each function and base point, find the first Taylor polynomial based at b and then use the Tangent Line Error Bound to find an interval J containing b so that the error bound is at most 0.01 on J.
   (a) $f(x) = \ln(x) \quad b = 1$.
   (b) $f(x) = \cos(x) \quad b = \frac{\pi}{6}$.
   (c) $f(x) = x^{\frac{1}{3}} \quad b = 8$.

3. In math 124, we used the tangent line approximation to estimate a function. For example problem 50, page 269 in Stewart says:

   > Suppose that we don't have a formula for $g(x)$ but we do know that $g(2) = -4$ and $g'(x) = \sqrt{x^2 + 5}$ for all x. Use a linear approximation to estimate $g(1.95)$ and $g(2.05)$.

   Use the Tangent Line Error Bound to bound the error in these two approximate values.