Homework #2 Math 126

These problems are based on the material in Sections 2 and 3 of the Taylor Notes.

1. Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at b and use the Quadratic Approximation Error Bound to bound the error $|f(x) - T_2(x)|$ on the interval I where
   (a) $f(x) = e^x \quad b = 0 \quad I = [-1, 1]$.
   (b) $f(x) = \ln(1 + x) \quad b = 0 \quad I = [-\frac{1}{2}, \frac{1}{2}]$.
   (c) $f(x) = \sin(x) \quad b = 0 \quad I = [-0.1, 0.1]$.
   (d) $f(x) = x^{\frac{1}{3}} \quad b = -8 \quad I = [-9, -7]$.

2. For each function and base point, find the second Taylor polynomial based at b and then use the Quadratic Approximation Error Bound to find an interval J containing b so that the error bound is at most 0.01 on J. Your intervals J should be bigger than the answers you obtained to problem 2 on Homework #1.
   (a) $f(x) = \ln(x) \quad b = 1$.
   (b) $f(x) = \cos(x) \quad b = \frac{\pi}{6}$.
   (c) $f(x) = x^{\frac{1}{3}} \quad b = 8$.

3. (a) Find the 5^{th} Taylor polynomial for $f(x) = x^5 - 3x^4 + 5x^2 + 6x - 2$ based at $b = 1$.

   Problem 3(a) gives a method for writing a polynomial in powers of $(x-1)$ instead of powers of x. It is sometimes useful if you are interested in the values of the polynomial near $x = 1$. Another technique would be to substitute $u = x - 1$ or $x = u + 1$, multiply out the polynomial, collect together powers of u and then substitute back $u = x - 1$. Using Taylor polynomials is a bit easier.

   (b) If f is the polynomial in part (a), why is the n^{th} Taylor polynomial equal to f whenever $n \geq 5$? Hint: See Taylor's Inequality. Can you make a similar statement for any polynomial?

4. (a) Find the n^{th} Taylor polynomial $T_n(x)$ for $f(x) = \ln(1 - x)$ based at $b = 0$.
   (b) Using Taylor's Inequality, find n so that $T_n(x)$ so that $|T_n(x) - \ln(1 - x)| < 0.01$ on the interval $[-0.5, 0.5]$.
   (c) On the smaller interval $I = [-0.1, 0.1]$, how close is T_n (from part (b)) to $\ln(1-x)$?
   (d)-(f) Repeat parts (a), (b), and (c) for the function $f(x) = \sqrt{1 + x}$ based at $b = 0$.