Homework #3 Math 126

These problems are based on the material in Section 4 of the Taylor Notes.

1. Write the sum in expanded form:

   (a) $\displaystyle\sum_{k=0}^{4} \frac{2k-1}{2k+1}$

   (b) $\displaystyle\sum_{k=1}^{5} \frac{1}{k} x^{2k}$

2. Write the sum using the sigma notation.

   (a) $\sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7}$

   (b) $1 + 2 + 4 + 8 + 16 + 32$

3. (a) Write the following in the form $\sum_{k=0}^{n} a_k x^k$ (i.e. combine and use the distributive law).

$$2\sum_{k=1}^{n} \frac{x^k}{k} - 3\sum_{k=1}^{n} 4^k x^k.$$

   (b) What is the second Taylor polynomial for this function (based at $b = 0$)?

4. (a) Find

$$\frac{d}{dx}\sum_{k=0}^{n} x^k.$$

   Express your answer using the sigma notation, and then in expanded form: $a_0 + a_1 x + a_2 x^2 + \ldots$, where a_0, a_1, and a_2 are given explicitly (only the first three terms are explicit).

   (b) Find

$$\int_0^t \left(\sum_{k=0}^{n} x^k\right) dx.$$

   Express your answer in two ways as in part (a).

5. Use Taylor's Inequality to show that for all x

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

There are two parts to this problem: first figure out what the n^{th} Taylor polynomial looks like, then use Taylor's Inequality to show that it converges to $\sin(x)$ for every x.

6. Use Taylor's Inequality to find n so that the n^{th} Taylor polynomial based at $b = 0$ for $\sin(x)$ satisfies

$$|T_n(x) - \sin(x)| < 10^{-3}$$

on the interval $[-\pi/2, \pi/2]$. Bound the error in this approximation on the smaller interval $[-\pi/4, \pi/4]$.

7. Use Taylor's Inequality to show that if $|x| < \frac{1}{2}$ then

$$-\ln(1-x) = \sum_{k=1}^{\infty} \frac{x^k}{k}.$$

Show that Taylor's Inequality cannot be used to prove that the series converges when $x > \frac{1}{2}$.