Homework #4 Math 126

These problems use the techniques of section 5 except for differentiation and integration of series. Each problem can be derived from the basic series given in Examples 4.2.

(a) In problems 1-6, find the Taylor series for $f(x)$ based at b. Your answer should have one Sigma (Σ) sign. On some problems you might want to describe the coefficients using a multi-part notation as in Example 5.5.
(b) Then write the solution in expanded form: $a_0 + a_1(x-b) + a_2(x-b)^2 + \ldots$ where you write at least the first three non-zero terms explicitly.
(c) Then give an interval I where the Taylor series converges.
Note that there are some hints below.

1. $f(x) = \cos(3x^2)$ based at $b = 0$.

2. $f(x) = \sin^2(x)$ based at $b = 0$.

3. $f(x) = e^{4x-5}$ based at $b = 2$.

4. $f(x) = \sin(x)$ based at $b = \frac{\pi}{6}$.

5. $f(x) = \dfrac{1}{4x-5} - \dfrac{1}{3x-2}$ based at $b = 0$.

6. $f(x) = \dfrac{x}{(2x+1)(3x-1)}$ based at $b = 1$.

7. The "sinh" and "cosh" functions are used, for example, in electrical engineering, and are defined by $\sinh(x) = (e^x - e^{-x})/2$, and $\cosh(x) = (e^x + e^{-x})/2$. Do questions (a) and (b) above for the function $h(x) = 2\sinh(3x) - 4\cosh(3x)$ based at $b = 0$.

8. Find the 6^{th} degree Taylor polynomial for $f(x) = \sin(3x-5)$ based at $b = 0$, without differentiating.

Hints:
Change the base from b to 0 by substituting $u = x - b$.
Be sure that the terms in your answers are numbers (coefficients) times powers of $x - b$.
Use the double angle formula in problem 2.
Use partial fractions in problem 6.
Use the addition formulae for $\sin(A \pm B)$ in problems 4 and 8.