Homework #5 Math 126

In the following problems, find the Taylor series for $f(x)$ based at b. Each series can be derived from the basic series given in Examples 4.2. Give an interval I where the Taylor series converges. In problems 1-8 as was done in Homework #4, use one Sigma (Σ) sign in your answer and then also write your answer in expanded form with the dot notation, including the first three not-zero terms. Note that there are some hints below.

1. $f(x) = -\ln(1-x)$ based at $b = 0$. (you might want to compare your answer to this problem and the solution obtained in HW#1 problem 4a.)

2. $f(x) = \dfrac{1}{(2x-5)^2(3x-1)}$ based at $b = 0$.

3. $f(x) = xe^x$ based at $b = 0$.

4. $f(x) = \ln(1+x^2)$ based at $b = 0$.

5. $f(x) = \begin{cases} \dfrac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0. \end{cases}$ based at $b = 0$.

   This function is used frequently in electrical engineering and signal processing. It is sometimes called $sinc(x)$ and pronounced "sink".

6. $f(x) = \int_0^x sinc(t)dt$ based at $b = 0$

   where the $sinc(x)$ function is given in problem 5. This integral is called the sinc integral function.

7. $f(x) = \int_0^x \tan^{-1}(t)dt$ based at $b = 0$.

8. $f(x) = \dfrac{2x}{1+x^2}$ based at $b = 0$.

   Do this problem in two ways: (a) Find the series for $1/(1+x^2)$ then multiply it by $2x$ and (b) Differentiate $\ln(1+x^2)$ and use problem 4. Your answer to (b) should agree with your answer to (a).

9. Find the fifth Taylor polynomial based at $b = 0$ for $f(x) = e^x \sin x$ by multiplication of the series for e^x and $\sin x$ (you do not have to find the general term of the product).

Hints:

- For problem 1, first find the series for f'. Note that this approach gives a larger interval I than obtained in the problems for section 3 (homework #3).

- In problem 2, find the partial fraction expansion by first finding the partial fraction expansion for $1/(2x-5)(3x-1)$, then multiply both sides of the equation by $1/(2x-5)$ and then expand $1/(2x-5)(3x-1)$ again. Now add the Taylor series for each of the three functions.
- For problem 3, if $T_n(x)$ converges to e^x then $xT_n(x)$ coverges to xe^x so you can multiply each term of the series for e^x by x.
- Problem 5: Notice that the series for $sinc(x)$ converges for all x and so as a function of x, it has a derivative which is found by differentiating the series term-by-term. So the *sinc* function is differentiable even at 0, in fact it has derivatives of all orders at 0.