

Math 126
Selected Taylor Notes HW Answers

Below are answers to some of the homework problems, and in some cases hints for getting there. These answers do not constitute full solutions of the problems, only the final result. You should supply all the intermediate steps necessary to deduce these results.

Homework # 1

- (a) $T_1(x) = 1 + x$, $|f(x) - T_1(x)| \leq (e/2)|x|^2 \leq e/2$.
- We'll give the answers in the form $|x - b| < c$ which is the same as $b - c < x < b + c$. In other words, the interval is $(b - c, b + c)$. As indicated in the Notes, the answers are not unique. You are expected to find some interval, not necessarily the best possible interval.
 - $T_1(x) = x - 1$ $|x - 1| \leq \sqrt{.005}$ or $|x - 1| \leq 0.07$.
 - $T_1(x) = 2 + \frac{1}{12}(x - 8)$. One interval that works: $|x - 8| < 1.4$. There is a bigger interval as well.
- $T_1(1.95) = -4.15$, $T_1(2.05) = -3.85$ and $|g(x) - T_1(x)| \leq .0008448$ for these values of x .

Homework # 2

- (a) $T_2(x) = 1 + x + \frac{x^2}{2}$, $|f(x) - T_2(x)| \leq (e/6)|x|^3 \leq e/6$.
- We'll give the answers in the form $|x - b| < c$ which is the same as $b - c < x < b + c$. In other words, the interval is $(b - c, b + c)$. (see the comment on problem 2 of Homework 1 above).
 - $T_2(x) = x - 1 - \frac{1}{2}(x - 1)^2$, $|x - 1| \leq (.03/8)^{\frac{1}{3}}$ or $|x - 1| \leq 0.155$.
 - $T_2(x) = 2 + \frac{1}{12}(x - 8) - \frac{1}{288}(x - 8)^2$. The interval $|x - 8| \leq 2.3$ will work.
- (a) $7 + 9(x - 1) - 3(x - 1)^2 - 2(x - 1)^3 + 2(x - 1)^4 + (x - 1)^5$.

Homework # 3

- (a) $-1 + \frac{1}{3} + \frac{3}{5} + \frac{5}{7} + \frac{7}{9}$.
- (a) $\sum_{k=3}^7 \sqrt{k}$.
- (a) $\sum_{k=1}^n kx^{k-1} = 1 + 2x + 3x^2 + \dots$ (or $\sum_{j=0}^{n-1} (j+1)x^j$)

5. $T_{2n+1}(x) = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!}$ and $|f^{(k)}(x)| \leq 1$, so that by Taylor's estimate $|\sin x - T_{2n+1}(x)| \leq |x|^{2n+2}/(2n+2)! \rightarrow 0$, by the lemma in section 4.
7. $f^{(n)}(x) = (n-1)!/(1-x)^n$, so $T_n(x) = \sum_{k=1}^n x^k/k$, and $|f^{(n+1)}(x)| \leq n!2^{n+1}$ on this interval, so that by Taylor's estimate, $|f(x) - T_n(x)| \leq \frac{1}{n+1}|2x|^{n+1} \rightarrow 0$, provided $|2x| \leq 1$.

Homework # 4

1. $\sum_{k=0}^{\infty} \left((-1)^k 3^{2k}/(2k)! \right) x^{4k} = 1 - \frac{9}{2}x^4 + \frac{27}{8}x^8 + \dots$, which converges for all x .
3. $\sum_{n=0}^{\infty} \left(e^3 4^n/n! \right) (x-2)^n = e^3 + 4e^3(x-2) + 8e^3(x-2)^2 + \dots$, which converges for all x .
5. $\sum_{k=0}^{\infty} \left(3^k/2^{k+1} - 4^k/5^{k+1} \right) x^k = \frac{3}{10} + \frac{59}{100}x + \frac{997}{1000}x^2 + \dots$, which converges on $I = (-\frac{2}{3}, \frac{2}{3})$.
7. $\sum_{n=0}^{\infty} \frac{c_n 3^n}{n!} x^n$ where $c_n = 2$ if n is odd and $c_n = -4$ if n is even, and in expanded form: $= -4 + 6x - 18x^2 + \dots$

Homework # 5

1. $\sum_{n=1}^{\infty} x^n/n = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$ which converges on $(-1, 1)$.
3. $\sum_{k=1}^{\infty} x^k/(k-1)! = x + x^2 + \frac{1}{2}x^3 + \dots$ which converges for all x .
5. $\sum_{k=0}^{\infty} (-1)^k x^{2k}/(2k+1)! = 1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 + \dots$ which converges for all x .
7. $\sum_{k=0}^{\infty} (-1)^k x^{2k+2}/\left((2k+1)(2k+2) \right) = \frac{1}{2}x^2 - \frac{1}{12}x^4 + \frac{1}{30}x^6 + \dots$, which converges on $(-1, 1)$.
9. $T_5(x) = x + x^2 + x^3/3 - x^5/30$.