Worksheet 5 Math 126

1. Suppose a train follows a circular path of radius R at constant speed. Compute the acceleration, and show that it is perpendicular to the direction of travel of the train and points into the circle.

2. Engineers in France observed that high–speed trains will not stay on the tracks if the tracks are constructed of straight lines and arcs of circles. Use the answer to problem 1 to explain why the trains will tend to leave the tracks at the juncture of a straight track and a circular track.

   Remark: If the acceleration of a vehicle changes abruptly, the passengers will feel a sudden push in the opposite direction. For instance, if the driver steps on the accelerator, you will be pushed backwards in your seat. Perhaps you have also felt a sideways jolt when riding in a train or car.

3. Suppose we want the train to make a smooth transition from a straight track along the line $y = -x$ for $x \leq -1$ to a straight track along the line $y = x$ for $x \geq 1$. That is, we seek a new curve $(x, y(x))$ between (-1,1) and (1,1) that will make the transition with no sudden changes in acceleration. What conditions are required on the function $y(x)$ so that it connects smoothly to the straight segments? (Your answer should be given in terms of the values of y and some of its derivatives at $x = \pm 1$).

4. In this problem you will construct a polynomial $y(x)$ that has the properties you listed in problem 3.
   (a) It's useful to look for an even polynomial, that is, one with only even powers of x. Why?
   (b) Find an even polynomial of degree 4 that has the properties you listed in step 3. (Degree 4 is suggested because you should have three conditions at $x = 1$, and an even polynomial of degree 4 has three coefficients.)

5. Find the curvature function κ for your curve, and check that it will vanish at both ends of the *curved* segment (and thus be continuous for the entire curve, including the straight segments).

   Remark: Modern train tracks use circular arcs to make a turn, but connect them to straight segments using transition tracks (called spirals). On the transition segments, the curvature increases from 0 at the end attached to the straight segment, to the curvature of the circular arc attached at the other end.