

UW Math Circle

Remember our expectations for Math Circle:

- **Be respectful.** Pay attention when an instructor or student is presenting, and be courteous to your fellow peers as you work together on math problems.
- **No electronics.** Please do not take out your phones outside of break time.
- **Be on time.** Please try to arrive before 5:00 and leave just after 6:30.

Week 1 – Games!

Try playing the following games with a friend and try to answer the following:

- *Will there always be a winner?*
 - *What is the best strategy for each game?*
 - *If both players use the best strategy, will the first or second player win?*
1. The game starts with a pile of 15 coins. Two players take turns removing either 1 or 2 coins from the pile. The player who takes the last coin loses.
 2. A rook starts in the top-right corner of a 8x8 chessboard. Two players take turns moving the rook either 1 or 2 squares left or down. The player to move the rook to the bottom-left square wins.
 3. Two stones are on a number line at positions 4 and 10. Two players take turns picking a stone and moving it either 1 or 2 spaces toward zero. Stones may not pass zero, pass each other, or land on the same number. The player to make the last legal move wins (leaving a stone on numbers 1 and 2).

4. Four stones are on a number line at positions 3, 6, 9, and 12. Two players take turns picking a stone and moving it either 1 or 2 spaces toward zero. Stones may not pass zero, pass each other, or land on the same number. The player to make the last legal move wins (leaving stones on numbers 1, 2, 3, and 4).

5. A rook starts in the top-right corner of an 8x8 chessboard. Two players take turns moving the rook either left or down as many squares as they'd like. The player to move the rook to the bottom-left square wins.

6. Four stones are on a number line at positions 3, 5, 8, and 13. Two players take turns picking a stone and moving it as many spaces as they'd like toward zero. Stones may not pass zero, pass each other, or land on the same number. The player to make the last legal move wins (leaving stones on numbers 1, 2, 3, and 4).

7. In this game, each player takes turns picking one of the numbers from 1 to 9. You can't pick a number that has already been picked earlier in the game. If one player's collection of numbers includes three numbers that add to 15, that player wins.

8. (bonus problem for those of you who know how to play chess) The game "two-move chess" is exactly like regular chess, except on each player's turn, they may make 1 or 2 legal moves. Prove that if white is playing perfectly, they will not lose the game.