Math Hour Olympiad June 20, 2010 Grades 6-7

Problem		
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Raise your hand when you are ready to discuss your solution to one (or several) of the problems. You have three discussion attempts for each problem; they will be recorded in the table above.

We kindly ask you to **turn off your cell phone** and keep it off until the end of the Olympiad.

- 1. Is it possible to draw some number of diagonals in a convex hexagon so that every diagonal crosses EXACTLY three others in the interior of the hexagon? (Diagonals that touch at one of the corners of the hexagon DO NOT count as crossing.)
- 2. A 3×3 square grid is filled with positive numbers so that
 - (a) the product of the numbers in every row is 1,
 - (b) the product of the numbers in every column is 1,
 - (c) the product of the numbers in any of the four 2×2 squares is 2.

What is the middle number in the grid? Find all possible answers and show that there are no others.

3. Each letter in HAGRID's name represents a distinct digit between 0 and 9. Show that

 $HAGRID \times H \times A \times G \times R \times I \times D$

is divisible by 3. (For example, if H=1, A=2, G=3, R = 4, I = 5, D = 6, then HAGRID $\times H \times A \times G \times R \times I \times D = 123456 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6$).

- 4. You walk into a room and find five boxes sitting on a table. Each box contains some number of coins, and you can see how many coins are in each box. In the corner of the room, there is a large pile of coins. You can take two coins at a time from the pile and place them in different boxes. If you can add coins to boxes in this way as many times as you like, can you guarantee that each box on the table will eventually contain the same number of coins?
- 5. Alex, Bob and Chad are playing a table tennis tournament. During each game, two boys are playing each other and one is resting. In the next game the boy who lost a game goes to rest, and the boy who was resting plays the winner. By the end of tournament, Alex played a total of 10 games, Bob played 15 games, and Chad played 17 games. Who lost the second game?