University of Washington Math Hour Olympiad, 2014
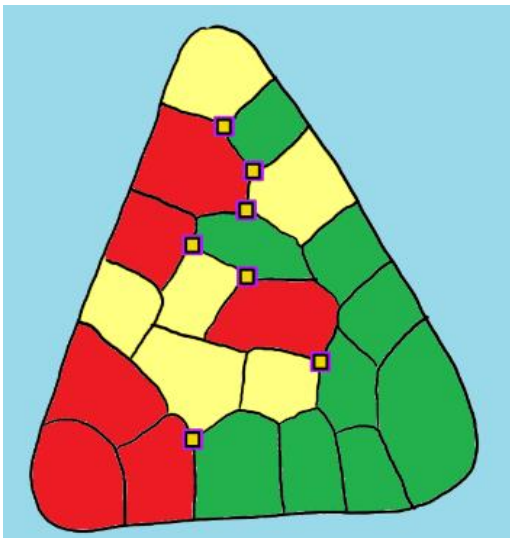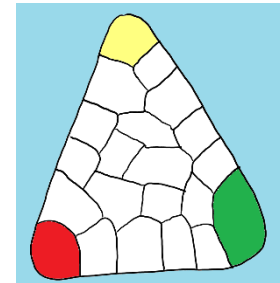
Grades 6-7

6. Hermione and Ron play a game that starts with 129 hats arranged in a circle. They take turns magically transforming the hats into animals. On each turn, a player picks a hat and chooses whether to change it into a badger or into a raven.

A player loses if after his or her turn there are two animals of the same species right next to each other. Hermione goes first. Who loses?

7. Three warring states control the corner provinces of the island whose map is shown to the right.

As a result of war, each of the remaining 18 provinces was occupied by one of the states. None of the states was able to occupy any province on the coast opposite their corner.

The states would like to sign a peace treaty. To do this, they each must send ambassadors to a place where three provinces, one controlled by each state, come together. Prove that they can always find such a place to meet.

For example, if the provinces are occupied as shown here, the squares mark possible meeting spots.