University of Washington Math Hour Olympiad, 2014

Grades 8-10

6. Homer goes on the 100-Donut Diet. A 100-Donut Diet Plan specifies how many of 100 total donuts Homer will eat each day. The diet requires that the number of donuts he eats does not increase from one day to the next. For example, one 5-day Donut Diet Plan is $40, 25, 25, 8, 2$.

Are there more 100-Donut Diet Plans with an odd number of days or plans where Homer eats an odd number of donuts on the first day?

7. If a is any number, $\lfloor a \rfloor$ is a rounded down to the nearest integer. For example, $\lfloor \pi \rfloor = 3$. Show that the sequence

$$\left\lfloor \frac{2^1}{17} \right\rfloor, \left\lfloor \frac{2^2}{17} \right\rfloor, \left\lfloor \frac{2^3}{17} \right\rfloor, \dots$$

contains infinitely many odd numbers.