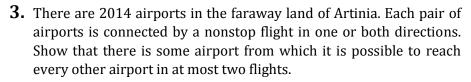# University of Washington Math Hour Olympiad, 2014

Grades 8-10

1. Sherlock and Mycroft are playing Battleship on a 4×4 grid. Mycroft hides a single 3×1 cruiser somewhere on the board. Sherlock can pick squares on the grid and fire upon them. What is the smallest number of shots Sherlock has to fire to guarantee at least one hit on the cruiser?

2.   A complete set of the Encyclopedia of Mathematics has 10 volumes. There are ten mathematicians in Mathemagic Land, and each of them owns two volumes of the Encyclopedia. Together they own two complete sets. Show that there is a way for each mathematician to donate one book to the library such that the library receives a complete set.

3. There are 2014 airports in the faraway land of Artinia. Each pair of airports is connected by a nonstop flight in one or both directions. Show that there is some airport from which it is possible to reach every other airport in at most two flights.

4. Hermione and Ron play a game that starts with 129 hats arranged in a circle. They take turns magically transforming the hats into animals. On each turn, a player picks a hat and chooses whether to change it into a badger or into a raven.

A player loses if after his or her turn there are two animals of the same species right next to each other. Hermione goes first. Who loses?

5.   An infinite number of lilypads grow in a line, numbered $\ldots, -2, -1, 0, 1, 2, \ldots$ Thumbelina and her pet frog start on one of the lilypads. She wants to make a sequence of jumps that will end on either pad 0 or pad 96. On each jump, Thumbelina tells her frog the distance (number of pads) to leap, but the frog chooses whether to jump left or right. From which starting pads can she always get to pad 0 or pad 96, regardless of her frog's decisions?