## University of Washington Math Hour Olympiad, 2015

## Grades 6-7

6. A casino machine accepts tokens of 32 different colors, one at a time. For each color, the player can choose between two fixed rewards. Each reward is up to $\$ 10$ cash, plus maybe another token. For example, a blue token always gives the player a choice of getting either $\$ 5$ plus a red token or $\$ 3$ plus a yellow token; a black token can always be exchanged either for $\$ 10$ (but no token) or for a brown token (but no cash). A player may keep playing as long as he has a token.

Rob and Bob each have one white token. Rob watches Bob play and win $\$ 500$. Prove that Rob can win at least $\$ 1000$.

7. Each of the 100 residents of Pleasantville has at least 30 friends in town. A resident votes in the mayoral election only if one of her friends is a candidate. Prove that it is possible to nominate two candidates for mayor so that at least half of the residents will vote.


