University of Washington Math Hour Olympiad, 2017

Grades 8-10

1. The Queen of Bees invented a new language for her hive. The alphabet has only 6 letters: A, C, E, N, R, T; however, the alphabetic order is different than in English. A word is any sequence of 6 different letters. In the dictionary for this language, the word TRANCE immediately follows NECTAR. What is the last word in the dictionary?



2. Is it possible to solve the equation $\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$ with *x*, *y*, *z* integers (positive or negative) such that one of the numbers *x*, *y*, *z* has one digit, another has two digits, and the remaining one has three digits?



3. The 10,000 dots in a 100×100 square grid are all colored blue. Rekha can paint some of them red, but there must always be a blue dot on the line segment between any two red dots. What is the largest number of dots she can color red?

The picture shows a possible coloring for a 5×7 grid.

4. Six flies rest on a table. You have a swatter with a checkerboard pattern, much larger than the table. Show that there is always a way to position and orient the swatter to kill at least five of the flies. Each fly is much smaller than a swatter square and is killed if any portion of a black square hits any part of the fly.



5. Maryam writes all the numbers 1-81 in the cells of a 9×9 table. Tian calculates the product of the numbers in each of the nine rows, and Olga calculates the product of the numbers in every column. Could Tian's and Olga's lists of nine products be identical?