University of Washington Math Hour Olympiad, 2019 Grades 6–7

6. A *triangulation* of a regular polygon is a way of drawing line segments between its vertices so that no two segments cross, and the interior of the polygon is divided into triangles. A *flip move* erases a line segment between two triangles, creating a quadrilateral, and replaces it with the opposite diagonal through that quadrilateral. This results in a new triangulation.



Given any two triangulations of a polygon, is it always possible to find a sequence of flip moves that transforms the first one into the second one?



7. Is it possible to place the numbers from 1 to 121 in an 11×11 table so that numbers that differ by 1 are in horizontally or vertically adjacent cells and all the perfect squares $(1, 4, 9, \ldots, 121)$ are in one column?