University of Washington Math Hour Olympiad, 2019 Grades 8–10

6. Hungry and Sneaky have opened a rectangular box of chocolates and are going to take turns eating them. The chocolates are arranged in a $2m \times 2n$ grid. Hungry can take any two chocolates that are side-by-side, but Sneaky can take only one at a time. If there are no more chocolates located side-by-side, all remaining chocolates go to Sneaky. Hungry goes first.



Each player wants to eat as many chocolates as possible. What is the maximum number of chocolates Sneaky can get, no matter how Hungry picks his?

7. There is a thief hiding in the sultan's palace. The palace contains 2019 rooms connected by doors. One can walk from any room to any other room, possibly through other rooms, and there is only one way to do this. That is, one cannot walk in a loop in the palace.

To catch the thief, a guard must be in the same room as the thief at the same time. Prove that 11 guards can always find and catch the thief, no matter how the thief moves around during the search.

