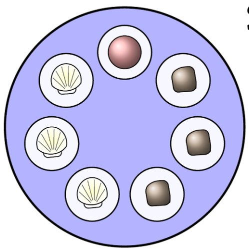
University of Washington Math Hour Olympiad, 2019

Grades 6–7

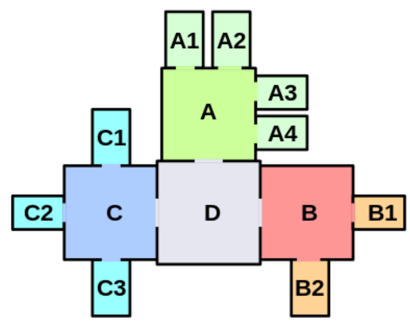
1. Three two-digit numbers are written on a board. One starts with 5, another with 6, and the last one with 7. Annie added the first and the second numbers; Benny added the second and the third numbers; Denny added the third and the first numbers. Could it be that one of these sums is equal to 148, and the two other sums are three-digit numbers that both start with 12?



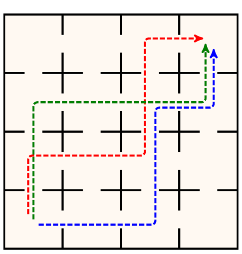
2. Three rocks, three seashells, and one pearl are placed in identical boxes on a circular plate in the order shown. The lids of the boxes are then closed, and the plate is secretly rotated. You can open one box at a time. What is the smallest number of boxes you need to open to know where the pearl is, no matter how the plate was rotated?

3. Two detectives, Holmes and Watson, are hunting the thief Raffles in a library, which has the floorplan exactly as shown in the diagram. Holmes and Watson start from the center room marked **D**. Show that no matter where Raffles is or how he moves, Holmes and Watson can find him. Holmes and Watson do not need to stay together.

A detective sees Raffles only if they are in the same room. A detective cannot stand in a doorway to see two rooms at the same time.



4. A museum has a 4×4 grid of rooms. Every two rooms that share a wall are connected by a door. Each room contains some paintings.



The total number of paintings along any path of 7 rooms from the lower left to the upper right room is always the same. Furthermore, the total number of paintings along any path of 7 rooms from the lower right to the upper left room is always the same.

The guide states that the museum has exactly 500 paintings. Show that the guide is mistaken.

5. The numbers 1–14 are placed around a circle in some order. You can swap two neighbors if they differ by more than 1. Is it always possible to rearrange the numbers using swaps so they are ordered clockwise from 1 to 14?