University of Washington Math Hour Olympiad 2023

Grades 8–10

Problem #1   Alex is on a week-long mining quest. Each morning, she mines at least 1 and at most 10 diamonds and adds them to her treasure chest (which already contains some diamonds). Every night she counts the total number of diamonds in her collection and finds that it is divisible by either 22 or 25. Show that she miscounted.

Problem #2   Hermione set out a row of 11 Bertie Bott's Every Flavor Beans for Ron to try. There are 5 chocolate-flavored beans that Ron likes and 6 beans flavored like earwax, which he finds disgusting. All beans look the same, and Hermione tells Ron that a chocolate bean always has another chocolate bean next to it. What is the smallest number of beans that Ron must taste to guarantee he finds a chocolate one?

Problem #3   There are 101 pirates on a pirate ship: the captain and 100 crew. Each pirate, including the captain, starts with 1 gold coin. The captain makes proposals for redistributing the coins, and the crew vote on these proposals. The captain does not vote.

For every proposal, each crew member greedily votes "yes" if he gains coins as a result of the proposal, "no" if he loses coins, and passes otherwise. If strictly more crew members vote "yes" than "no," the proposal takes effect.

The captain can make any number of proposals, one after the other. What is the largest number of coins the captain can accumulate?

Problem #4   There are 100 food trucks in a circle and 10 gnomes who sample their menus. For the first course, all the gnomes eat at different trucks. For each course after the first,

> gnome #1 moves 1 truck left or right and eats there;
> gnome #2 moves 2 trucks left or right and eats there;
>   ...
> gnome #10 moves 10 trucks left or right and eats there.

All gnomes move at the same time. After some number of courses, each food truck had served at least one gnome. Show that at least one gnome ate at some food truck twice.

Tesla   Edison

Problem #5   The town of Lumenville has 100 houses and is preparing for the math festival.

The Tesla wiring company lays lengths of power wire in straight lines between the houses so that power flows between any two houses, possibly by passing through other houses.

The Edison lighting company hangs strings of lights in straight lines between pairs of houses so that each house is connected by a string to exactly one other.

Show that however the houses are arranged, the Edison company can always hang their strings of lights so that the total length of the strings is no more than the total length of the power wires the Tesla company used.