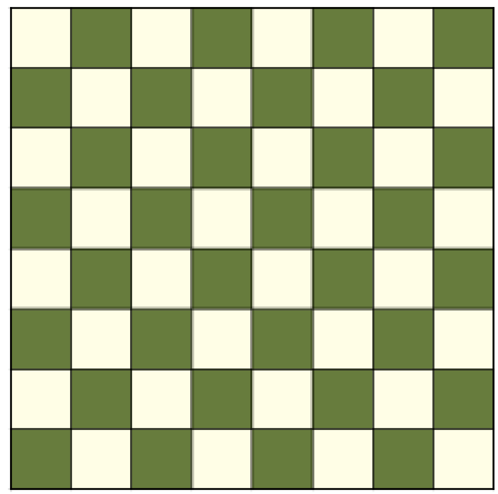
MATH TILINGS at Math Hour!

Inspired by Activities of Julia Robinson Math Festival and Nina Cerutti and Leo B. of SFMC.

Julia Robinson Mathematics Festival

Directed By Joshua Zucker

Tiling Torment



The problem

There are many problems that involve tiling (covering) all the squares on a chessboard (or similar board) with tiles of various sizes. The chessboard may be 8x8, 7x7 or other sizes and may or may not have squares missing. The tiles can be dominoes (2x1) or tiles of other sizes.

Questions

The Basics

1. Is it possible to tile a 7x7 board with 2x1 tiles?

2. In general, is it possible to tile an nxn board with 2x1 tiles? If so, which boards can you tile and why?

Taking it Further

3. Now consider the 7x7 board again. If you remove one square, is it possible to tile the board? If so, does it matter which square you remove? Describe completely.

4. In general, if n is odd, is it possible to tile an nxn board with 2x1 tiles if one square is covered with a 1x1 tile? Does it matter which square is covered?

5. Remove two diagonally opposite corners of a chessboard. Is it possible to tile this shape with 31 2x1 tiles?

6. In general, if n is even, is it possible to tile an nxn board with 2x1 tiles if two squares are removed? Does it matter which two squares are removed?

7. Is it possible to tile an 8x8 board with 21 “L-shaped” tiles of three squares and one 1x1 tile? If so, how? Describe all possible locations for the 1x1 tile. If not, why not?

Dr. Brandy Wiegers wiegersb@cwu.edu

2-Dimensional Tiling…

When we tile something in 2-D using our special Tiles, we care about their height of 2^i and their width of 2^i. This total square area must be used to tile a square part of the rectangle we are trying to tile over.

Consider a rectangle of dimensions 9 and 10.

a. What is the greatest number of Tiles of size 2^i x 2^i can you use to tile this rectangle?
b. What is the least number of Tiles you can use to tile this rectangle?

RULE OF 1

Suppose you are only allowed to use exactly one Tile of each size. What sizes of rectangles can you tile with this new condition?

Fill in the chart on the following page with a **1** on the rectangles that Rule of 1 allow you to create. *For example if you have a 4x4 shaped rectangle you can use 1 of the 2^2x2^2 Tiles to tile this rectangle shape.*

RULE of 2

Now suppose you are only allowed to use up to two squares of each size. What types of rectangles can you tile with this condition? To keep track of your work (or to see some patterns), use the table provided.

For example if you have a 4x6 shaped rectangle you can use 1 of the 2^2x2^2 Tiles and 2 of the 2^1x2^1 to tile this rectangle shape.

Questions about 2-D Tilings…

1. What patterns do you see in the table?
2. If you want to tile a rectangle of dimensions h x w what rectangles can you tile given Rules 1 and 2?

Further exploration – 3D Tiling…