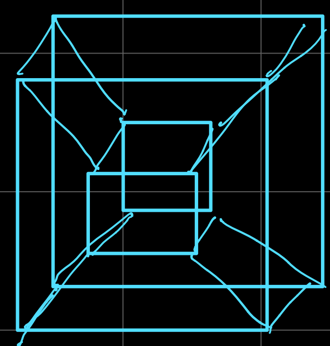
$\sqrt{2}$ e

1 π

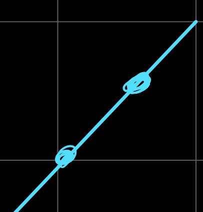
$3/4$

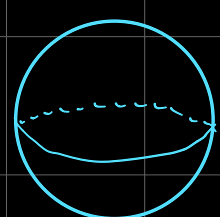
Numbers

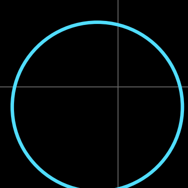
and



Shapes







What numbers are...

0

e 1 2

8 3 π

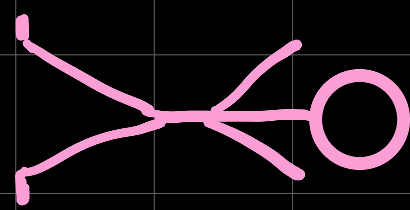
42

-2

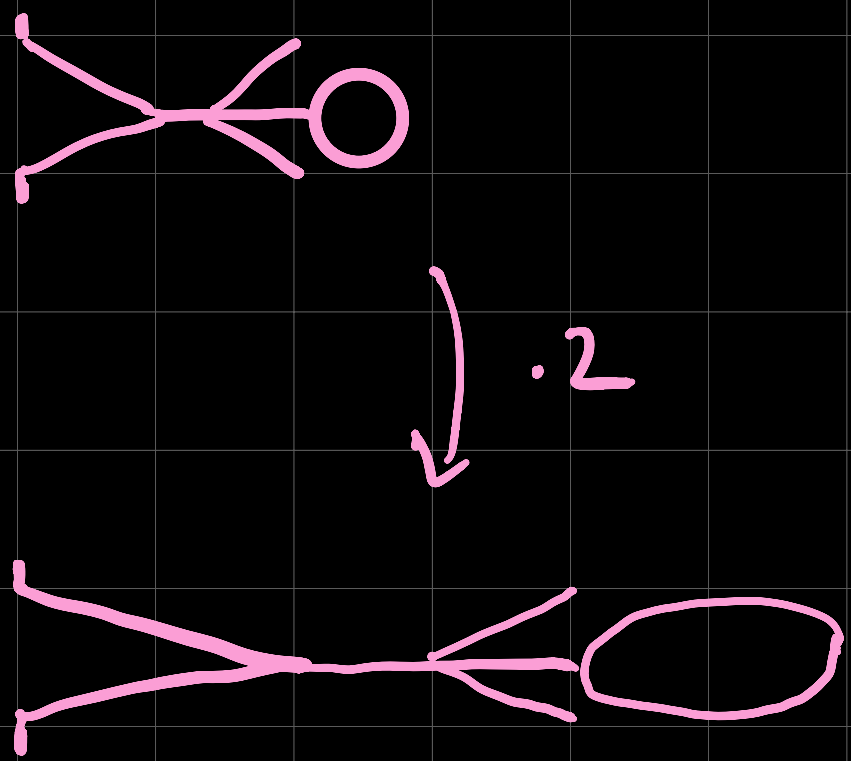
$\frac{1}{2}$ 0.123

$+$, $\cdot$, $\div$ ....

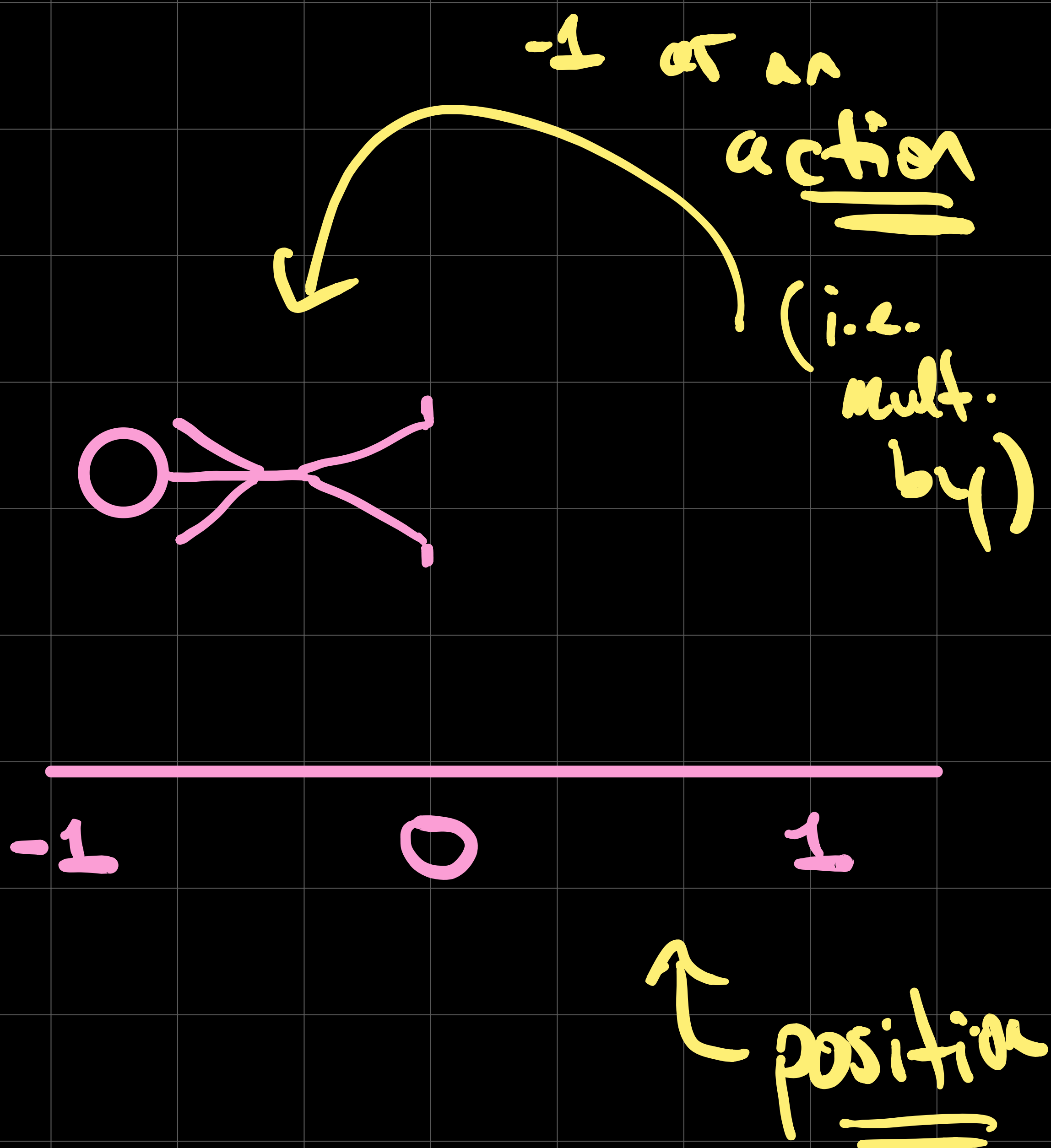
What numbers *do* . . .



What numbers <u>do</u>...

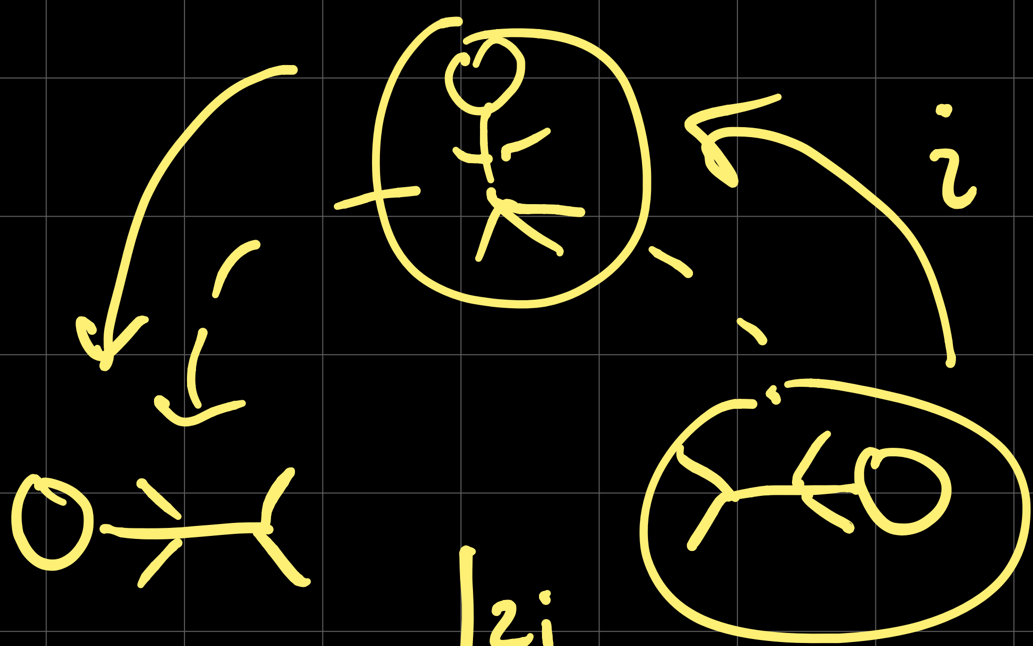


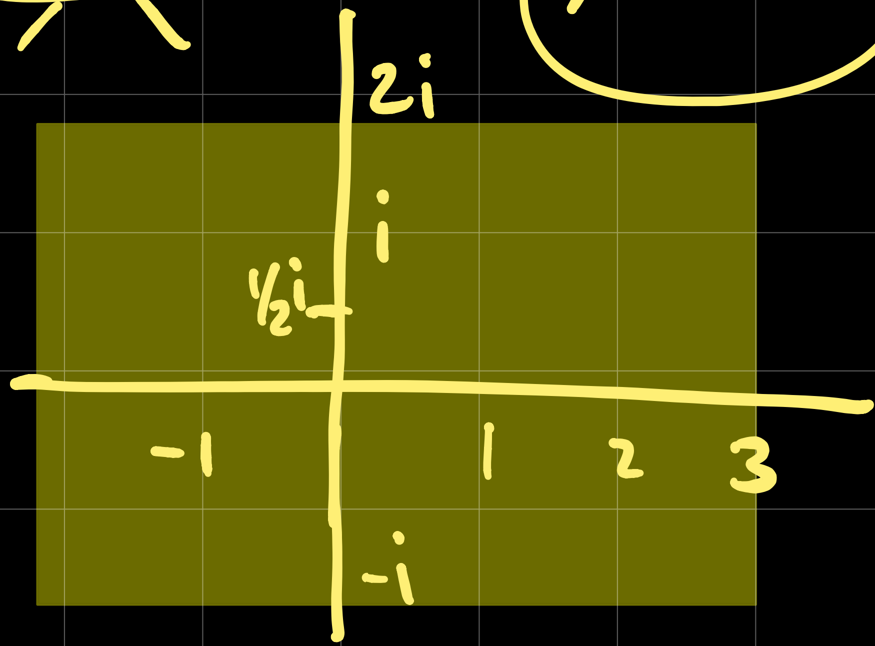
What numbers <u>do</u> ...



-3 -2 -1 0 1 2 3 4

What motion, if done twice, does what (−1) does?





Complex Numbers

$$i \cdot i = -1$$

$$2i \cdot 3 = 6i$$

$$(2+i) \cdot i = 2i + i^2$$
$$= 2i - 1$$

Complex numbers...

$$(1+3i)\cdot(5+7i) = 1\cdot(5+7i) + 3i(5+7i)$$

$$= 5+7i+15i+21i^2$$

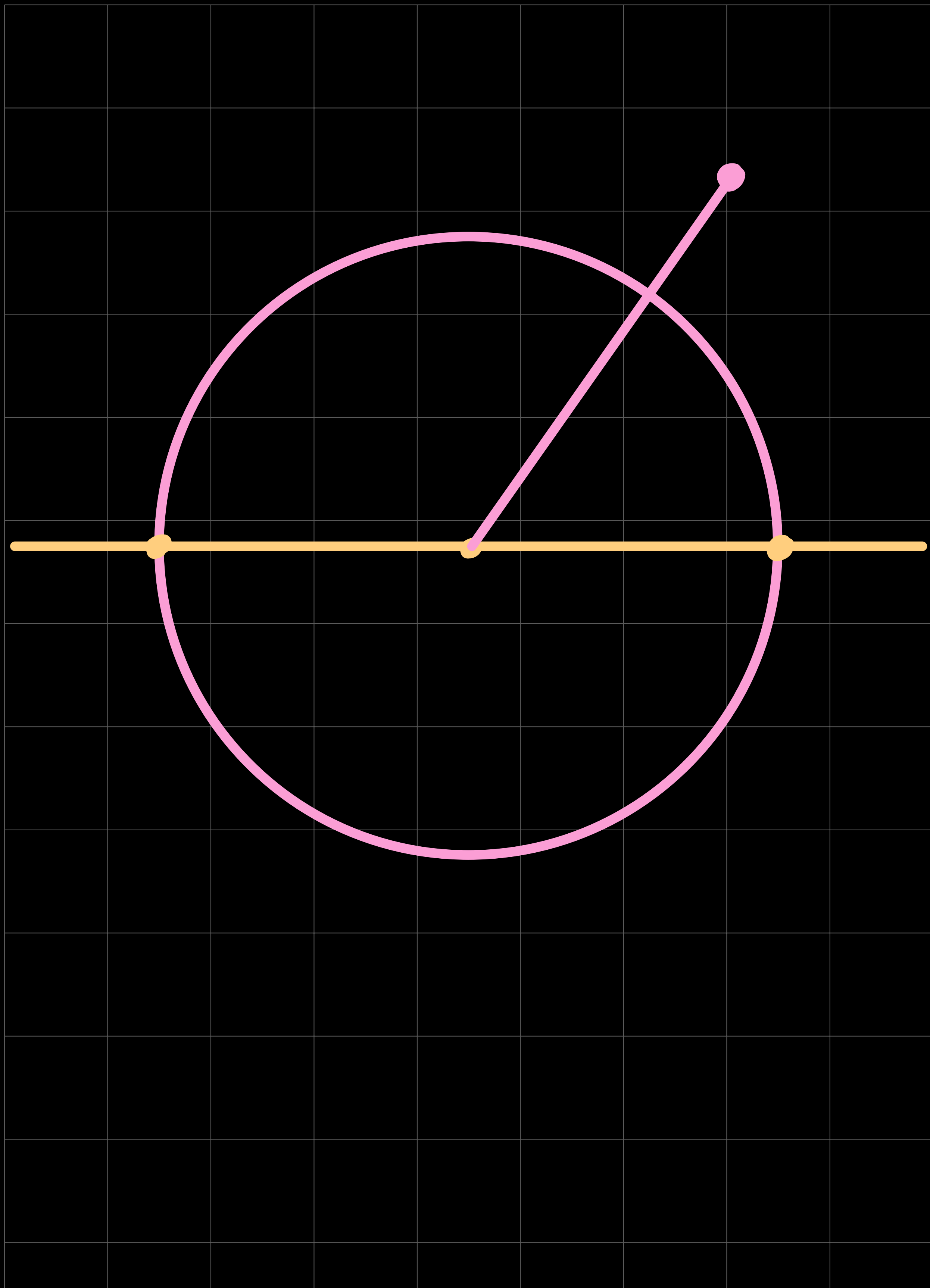
$$= (5-21) + 22i$$

$$= -16 + 22i$$

$$(1+i)(1-i) = 1 \cancel{-i+i} + i(-i)$$

$$= 1+1 = 2$$

$$\frac{1}{1+i}\frac{(1-i)}{(1-i)} = \frac{1-i}{2}$$



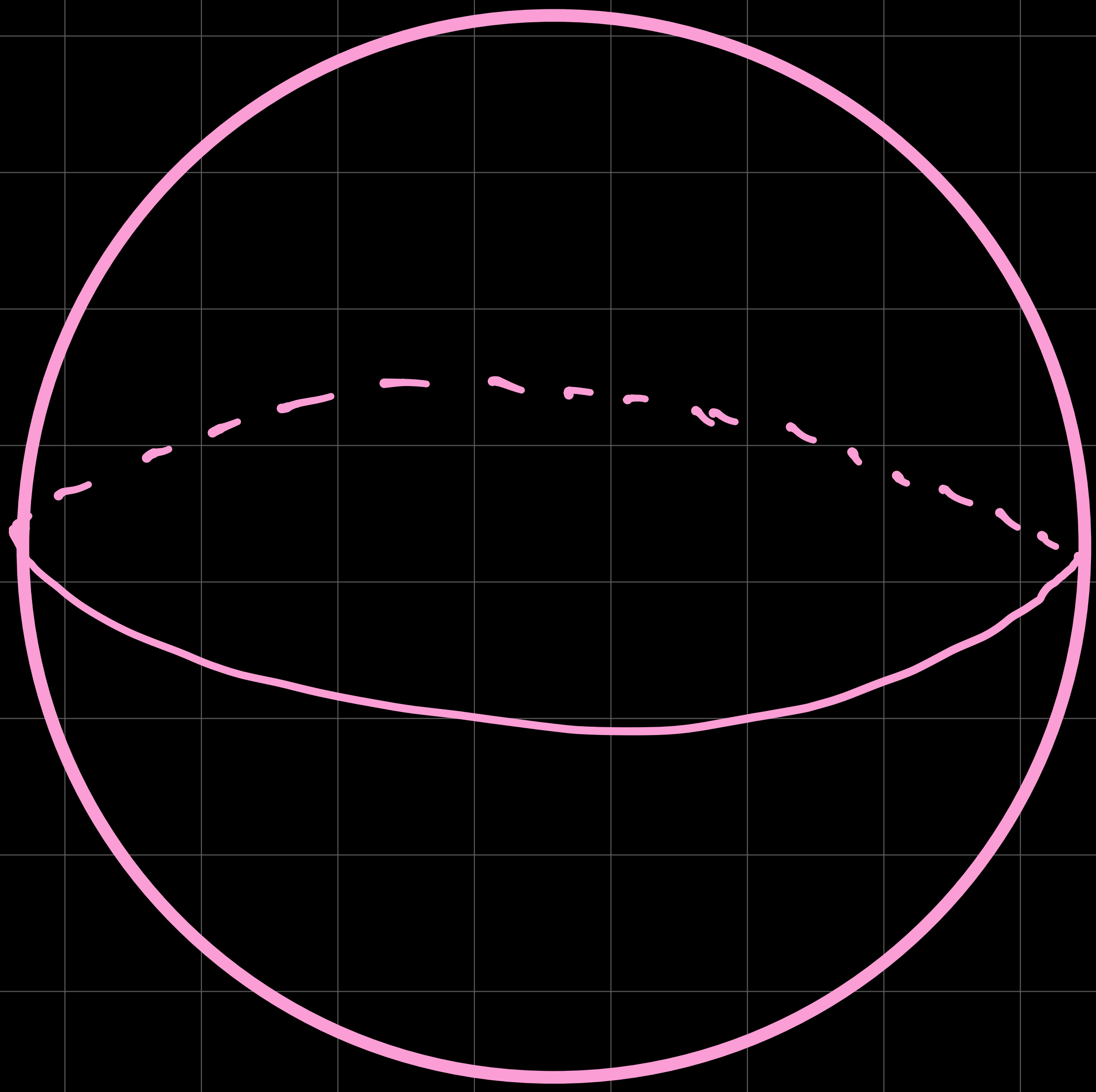
Complex #s Review:

- Multiplying rotates and scales

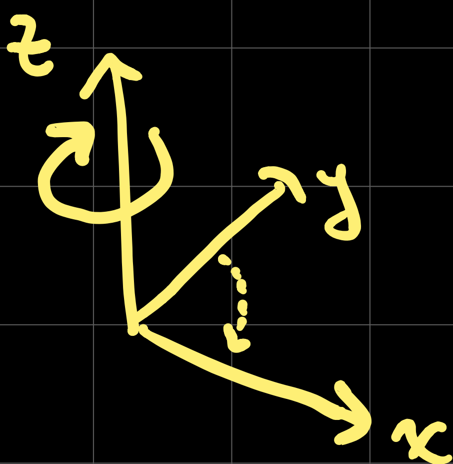
- a cpx # of length 1 can be thought of as...

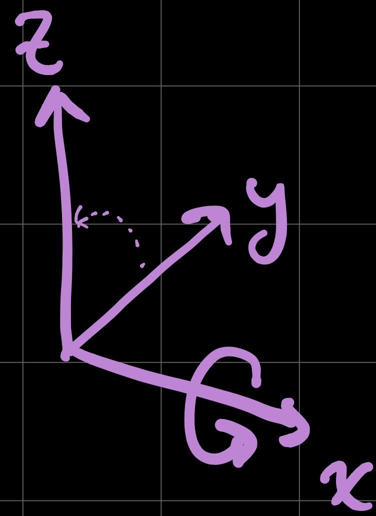
  └ a rotation

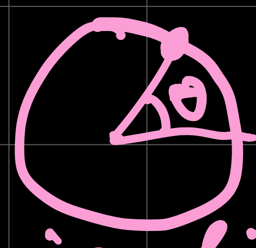
  └ a position on the circle.



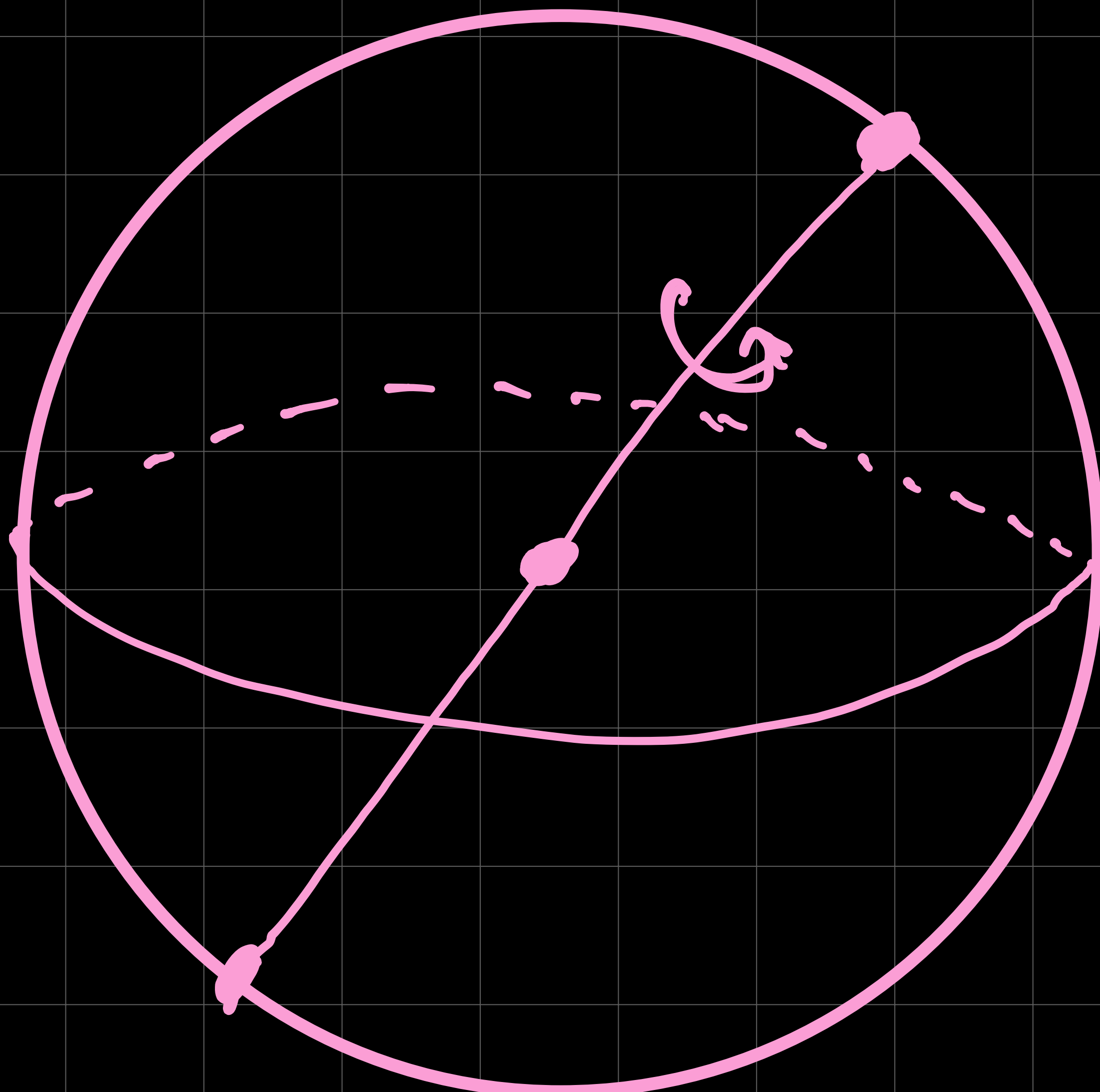
. . . Stand up!







1 piece of info!



\# of pieces of info to describe a pt. on the sphere = 2

\# ______ ... a rotn? = 3

New numbers *are* ...

$\pm 1 \cdot$ length to scale by

$i^2 = -1$

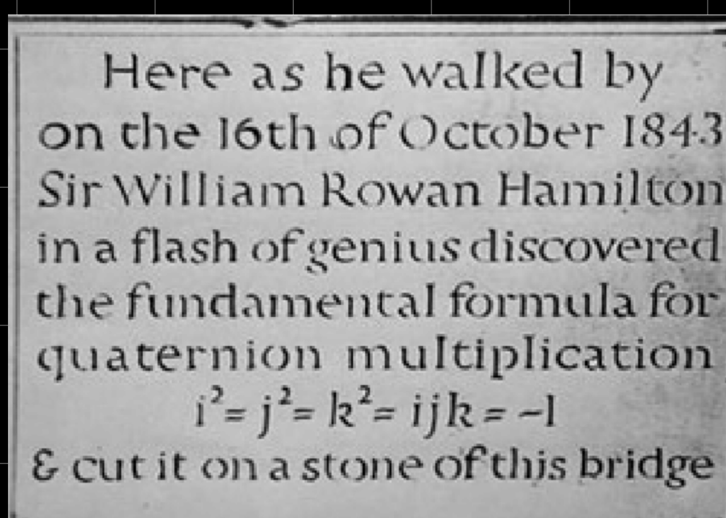
$j^2 = -1$

$ij = k$

$ij = -ji$

$$\left(3+2i+7j+6k\right) \cdot \left(2+\frac{1}{2}i+\dots\right)$$

$$= \dots$$


Here as he walked by
on the 16th of October 1843
Sir William Rowan Hamilton
in a flash of genius discovered
the fundamental formula for
quaternion multiplication
$i^2 = j^2 = k^2 = ijk = -1$
& cut it on a stone of this bridge

New #'s: quaternions

$ij = k$
$jk = i$
$ki = j$

$$a + bi + cj + dk$$

New numbers **do**?

- **position** in 3-d space,

  recorded as

  $\underline{x}i + \underline{y}j + \underline{z}k$

- **actions** correspond

to quaternions

$q = a + bi + cj + dk$

$$xi + yj + zk$$

$$q \cdot (xi + yj + zk) \frac{1}{q}$$

Real #'s ... $\mathbb{R}$

1

Complex #'s ... $\mathbb{C}$

2

Quaternions ... $\mathbb{H}$

4

Octonions ... $\mathbb{O}$ 8

$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
1	2	4	8

... what's next?

Thm (Bott, Kervaire-Milnor)

There is no "arithmetic of n-tuples"*

except when

$n = 1, 2, 4, 8.$

* division algebra