We will start our study of Combinatorics by discussing the notion of a **graph**. A graph consists of points, called *vertices*, some of which are connected by line segments, called *edges*. We usually assume that

- NO vertex is connected to itself, that is, there are no *loops*, and that
- NO two vertices are connected by more than one edge.

In the literature such graphs are somitemes called *simple graphs*. The number of edges that start at a given vertex is called the *degree* of that vertex. The following theorem we will prove in class is very useful for counting edges.

Theorem 1 *Let G be a graph on n vertices whose degrees are $d_1, d_2, \ldots, d_n$. Then the number of edges of G equals $(d_1 + d_2 + \cdots + d_n)/2$.*

Since the number of edges is an integer, it follows that the sum $\sum_{i=1}^{n} d_i$ of all the degrees is an *even* number. Hence we have the following corollary.

Corollary 2 *In a graph, the number of odd-degree vertices is even.*