

- (a) How many diagonals does an  $n$ -gon have?

Diagonals of an  $n$ -gon form a graph on  $n$  vertices with degree of each vertex being equal to  $(n - 3)$  (it is connected to all the remaining  $(n - 1)$  vertices except for its two neighbors in the  $n$ -gon). Thus the total number of diagonals is the number of edges in this graph, which is equal to  $n(n - 3)/2$ .

- (b) Can a kingdom in which seventy seven roads lead out of each city have exactly 1000 roads? No. For such a kingdom to have 10000 roads, there would have to be  $V$  cities, where  $V$  satisfies  $77 \cdot V = 2 \cdot 10000 = 20000$ . But this is impossible since 20000 is not divisible by 77.