

Reading assignment: Read carefully §§9.2 and 10.1, and p. 220 of §10.3.

Written assignments:

1. p. 200, #(33) How many Hamiltonian cycles does K_n have?
2. p. 200, #(34) Find the number of Hamiltonian cycles of $K_{m,n}$.
Note for the first two problems: make sure to read the note preceding #(33) in the book. It explains when two Hamiltonian cycles are considered different.
3. p. 200, #(41) Prove that the statement of exercise #(40) is not true if we only assume that $d_x + d_z \geq n - 1$.
4. There are 100 towns in a country and some of them are connected by airlines. It is known that one can reach every town from any other (perhaps with several intermediate stops). Prove that you can fly around the country and visit ALL the towns making no more than 198 flights.
Hint: Consider a spanning tree of the graph of routes. Double each edge of this tree. Apply Euler's theorem to the resulting multigraph.
5. Let G be a connected graph with at least two vertices. Prove that it has a vertex such that if this vertex is removed (along with all edges incident with it), the remaining graph is also connected.
Hint: Consider a spanning tree and one of its leaves.
6. Prove that if a tree has a vertex of degree d , then it has at least d leaves.
Hint: mimic the proof of Theorem 10.5 starting your walk from a vertex of degree d .