

Problem Set 3

514 - Networks and Combinatorial Optimization

Autumn 2023

Exercise 3.2 (10pts)

- (a) Consider $P := \{x \in \mathbb{R}^n \mid Ax = b, x \geq \mathbf{0}\}$ for $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$. Fix a point $y \in P$ that minimizes $|\text{supp}(y)|$ where $\text{supp}(y) := \{j \in \{1, \dots, n\} \mid y_j \neq 0\}$. Prove that $|\text{supp}(y)| \leq m$.
- (b) Let $X \subseteq \mathbb{R}^n$. Prove that for any $x \in \text{conv}(X)$, there is a subset $X' \subseteq X$ with $|X'| \leq n + 1$ so that $x \in \text{conv}(X')$.

Exercise 3.3 (10pts)

Let $A \in \mathbb{R}^{m \times n}$ and let $b \in \mathbb{R}^m$ with $m \geq n + 1$. Suppose that $Ax \leq b$ has no solution x . Prove that there are indices i_0, \dots, i_n so that the system $A_{i_0}^T x \leq b_{i_0}, \dots, A_{i_n}^T x \leq b_{i_n}$ has no solution x .

Remark. The last exercise is taken from A. Schrijver's 2009 lecture notes.