

## Problem Set 5

**CSE 531 - Computational Complexity**

Winter 2024

**Exercise 4.12 (from the book of Arora and Barak; 10pts)**

Define  $\text{polyL} := \bigcup_{c>0} \text{DSPACE}((\log n)^c)$ . Steve's Class **SC** (named after Steve Cook) is defined to be the set of languages  $L \subseteq \{0,1\}^*$  that can be decided by a deterministic Turing machine that runs in polynomial time and  $(\log n)^c$  space, for some constant  $c > 0$ . It is an open problem whether  $\text{PATH} \in \text{SC}$ . Why does Savitch's Theorem not resolve this question? Is **SC** the same as  $\text{polyL} \cap \text{P}$ ?

**Remark.** This question should be understood as being somewhat open ended.

**Exercise 2 (10pts)**

Consider the language

$$\text{BIPARTITE} = \left\{ G \mid \begin{array}{l} G = (V, E) \text{ is undirected graph and} \\ \exists \text{ partition } V = V_0 \dot{\cup} V_1 : \text{ all edges in } E \text{ run between } V_0 \text{ and } V_1 \end{array} \right\}.$$

Prove that  $\text{BIPARTITE} \in \text{NL}$ .

**Hint.** First prove that  $G \in \text{BIPARTITE}$  if and only if  $G$  does not contain an odd length cycle. Next, make use of a theorem we have proven in class.