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SUPPLEMENTARY BIBLIOGRAPHY AND COMMENTS
FOR THE RUSSIAN EDITION OF "CONVEX ANALYSIS" (1972)

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This book was written in 1967, although it did not appear in print until the beginning of 1970. Since that time, many papers have been published on convexity and its role in problems of optimization. In the accompanying bibliography, I have tried to list with some degree of completeness the new papers that deal directly with conjugate convex functions, subgradients, dual convex programs, perturbations of constrained optimization problems, Lagrange multipliers, minimax theory, convex processes, and closely related topics. Also cited are several older papers, mostly in the Russian literature, which previously escaped my attention. I have indicated some of the research now being done in applying conjugate convex functions and duality in such areas as approximation, economics, statistical decision, and optimal control, but here the coverage is far from exhaustive. No attempt has been made to cover what has been happening in other important and active areas of convex analysis, for example the theory of extreme points.

Several new expository works are available: the book of Stoer and Witzgall [166], treating in finite-dimensional spaces the theory of linear inequalities, convex sets, conjugate convex functions and convex programs; the article of Ioffe and Tikhomirov [75], explaining various facets of the infinite-dimensional theory of conjugate convex functions; two monographs of Gol'shtein [54,55], developing a duality theory for infinite-dimensional convex programs which is based on classical minimax theorems, rather than conjugate convex functions; and the lecture notes of Rubinshtein [163], likewise dedicated to applications of convexity to optimization, and likewise without discussion of conjugacy.

A paper of Klee [84] surveys the sharpest facts that are known about the separation of convex sets. Some other papers dealing essentially with separation properties are: Klee and Olech [84], Maury [106], Rubinshtein [159,162] and Witzgall [190].

Further results on the continuity of convex functions have been obtained by Asplund and Rockafellar [3], Castaing [15], Kruskal [90] and Lescarret [96]. The paper of Asplund and Rockafellar also treats differentiability properties in relationship to continuity and the convergence of minimizing sequences, as well as the Legendre transformation in infinite-dimensional spaces. The almost-everywhere differentiability of finite convex functions on $R^n$ has been generalized in a certain sense by Asplund [2] to Banach spaces (see Trojanski [172] in connection with Asplund's hypotheses). Reshetniak [131], using the theory of distributions, has obtained new results on twice-differentiability on $R^n$. For work on the special gradient mappings called "proximations" (e.g. projections on convex sets), see Aris [1], Castaing [16], Kruskal [90], Lescarret [95, 97], Moreau [110, 113] and Zarantonello [193].

The connection between subdifferential mappings and the theory of nonlinear monotone operators and variational inequalities is discussed at length in an expository article of Rockafellar [137]; see also [139, 140, 142]. Applications to partial differential operators have been explained by Brezis [11]. Much effort has been devoted to obtaining formulas for constructing the subgradients and directional derivatives of convex functions in various situations; see Brøndsted [12], Dubovitskii and Miliutin [30,31], Gol'shtein [52,55], Hogan [68], Ioffe [71], Ioffe and Levin [73], Ioffe and Tikhomirov [75], Levin [99,101,102], Moreau [112], Pshenichnyi [127], Rockafellar [151] and Valadier [176,177].

Convex functionals defined by integrals have also been studied extensively, due to their frequent occurrence in infinite-dimensional optimization problems, including many problems of a probabilistic nature. A versatile theory is now available. Some of the central results are described in a recent survey paper of Rockafellar [150]. For more results and details, also on the closely related subject of the measurability of convex set-valued mappings, consult Castaing [14,15,16,17], Castaing and Valadier [18], Goodman and Hoffman-Jorgensen [57], Hukuhara [70], Ioffe [72], Olech [120], Rockafellar [138,149,151], Valadier [175,177,178,179,180], and Van Cutsem [181]. The recent paper of Valadier [180] is recommended as a reference for the known measurability results in infinite-dimensional spaces.

Integral functionals have been used in defining continuous analogues of the operations of addition and infimal convolution of convex functions; see Ioffe and Tikhomirov [74, 75] and Valadier [178]. Other research on infimal convolution has been carried out by Maury [106] and Moreau [111].

The duality theory for convex programs presented in this book has been generalized to infinite-dimensional spaces by Joly and Laurent [79] and Rockafellar [136] (see also [146]). Further duality results for convex programming problems may be found in the work of Astaf'ev [4], Auslender [5], Brans and Claesen [9], Breckner and Kolumbán [10], Geoffrion [48], Gol'shtein [50,51,53,54,55], Kaplan and Rubinshtein [81], Karamardian [81], Moreau [113], Neustadt [119], Pshenichnyi [127,128], Roffin [129,130], Ritter [132], Rockafellar [148], Rubinshtein [158,161,162,163], Stoer and Witzgall [166], Walkup and Wets [183], Whinston [187] and Yamasaki [192]. For linear programs in infinite-dimensional spaces see, besides many of the papers just cited, Kallina and Williams [80], Krabs [86,87], and for a particularly interesting class of examples Grinold [62] (and the references given there). Special cases such as "geometric" (exponential) programs have been treated by Duffin [33], Hamala [65], Peterson [121], Peterson and Ecker [122,123] and Rockafellar [143]. Fiacco and McCormick [39] furnish dual interpretations of penalty methods of computation, while Everett [36], Falk [37], and Gould [59] have discussed the ordinary dual program in the nonconvex case. Moreau [111] and Weiss [184] have defined conjugate correspondences for nonconvex functions.

Minimax problems dual to each other have been studied by Lebedev and Tynjanski [94], McLinden [109], Rockafellar [135], and Tynjanskii [174]. The dissertation of McLinden also develops for the first time a duality theory of operations such as addition, extremal convolution, and forming images under linear transformations, in the case of saddle functions. The theory of conjugate equivalence classes of closed saddle functions has been extended to infinite-dimensional spaces by Rockafellar [146], and the subdifferential mappings associated with such functions have been analyzed by Gossez [58] and Rockafellar [141]. Applications to Hamiltonian dynamical systems may be seen in papers of Daures [25] and Rockafellar [145,153].

The continuous behavior of the optimal value and optimal solutions in an extremum problem with respect to various (non-convex) perturbations has been investigated by Dantzig, Folkman and Shapiro [24], Evans and Gould [35], Gol'shtein and, Movshovich [56] (see also [54,55]), and Krabs [88], and in a more general way, in terms of topologies for spaces whose elements are convex sets or functions, by Joly [78] and Mosco [114,115]. Measurability of the dependence of the optimal value and optimal solutions on parameters, as is a concern especially in stochastic problems, follows from results on measurable set-valued mappings cited above (e.g. [138] and [181]). The marginal value theorem of Williams [188], describing the directional derivatives of the optimal value in a linear programming problem with respect to "smooth" perturbations, has been generalized by Gol-shtein [54,55] to a much broader result, applicable not only to ordinary convex programming problems, but also to a large class of other problems whose solutions can be represented in terms of saddle points. For additional nonclassical results on one-sided directional derivatives in optimization problems, see Danskin [23], Demyanov [26,27], Hogan [68], Pshenichnyi [128], Sotskov [164,165], and Tsvetanov [173]. Such results are intimately related to the study of necessary conditions for an extremum in nonconvex problems, in particular the theorems on Lagrange multipliers that have been developed for application to optimal control. There is a huge literature on this subject, which cannot be reviewed here, but the following papers, besides those already cited in connection with one-sided directional derivatives and the theory of subgradients of convex functions, include most of the strongest known results of a general nature and provide further references: Canon, Cullum and Polak [13], Dubovitskii and Miliutin [30,31], Gamkrelidze [44], Gamkrelidze and Kharatishvili [45], Gould and Tolle [60], Guignard [61], Halkin [63], Halkin and Neustadt [64], Hestenes [67], Mangasarian [105], Nagahisa and Sakawa [116], Neustadt [117,118,119], Pshenichnyi [125,128], Ritter [133], and Varaiya [182]. For second-order conditions in $R^n$, see for example Fiacco [38] and McCormick [108].

Direct applications of the theory of conjugate convex functions to optimal control and the calculus of variations, including the derivation of necessary and sufficient conditions for a minimum in problems of "convex" type, have been pursued recently by Bittner [8], Heins and Mitter [66], Ioffe [72], Ioffe and Tikhomirov [74,75], Rockafellar [144,145,147,152,153], Temam [167,168,169], and Tsvetanov [170,171,173]. In some closely related work of Makarov and Rubinov [104] and Rubinov [155], motivated by mathematical economics, variational problems involving "convex processes" have been studied.

The general theory of convex processes has itself been studied by Robinson [134] and Rubinov [154,156,157]. For some further applications of duality theory to mathematical economics, see Balinski and Baumol [6], Gale [43] and Williams [189].

New applications of convex analysis to approximation theory have been the subject of much research; we cite in particular Duffin and Karlovitz [34], Garkavi [46], Gol'shtein [52,55], Ioffe and Tikhomirov [75], Joly and Laurent [79], Laurent and Pham-Dinh-Tuan [91], Levin [100], and Pshenichnyi [128]. Levin's paper emphasizes consequences of Helly's theorem on intersections of convex sets, and it deals with additional topics besides approximation.

In statistics, striking use of the theory of conjugate convex functions has been made by Barndorff-Nielsen [7]. The papers of Dieter [29] and Krafft [89] point to areas in which many more applications are possible. A duality theorem in stochastic programming has been developed by Wets [185]. For dual problems connected with the Neymann-Pearson lemma, consult Francis and Wright [41].

Some examples of dual convex programs in science and engineering have been described by Duffin [32]. Yamasaki [192] has indicated examples in potential theory. Interesting examples in the theory of analytic functions may be found in Khavinson [83] and Lax [93].

The role of convexity in the computation of solutions to optimization problems has been discussed recently by Wolfe [191], among others. Related computational material may be found in works such as Auslender [5], Céa [19], Cherrault and Loridan [20], Cruceanu [21], Daniel [22], Demyanov and Rubinov [27,28], Fiacco and McCormick [39], Folkman and Shapiro [40], Geoffrion [47], Lasdon [92], and Polak [124]. For recent work on optimization problems in networks, see Frank and Frisch [42], Hu [69] and Iri [77].

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