

# Reconstruction of subdomain boundaries of piecewise constant coefficients of the Radiative Transfer Equation from Optical Tomography data.

S. R. Arridge<sup>\*</sup>, O. Dorn<sup>†</sup>, J.P. Kaipio<sup>‡</sup>, V. Kolehmainen<sup>‡</sup>, M. Schweiger<sup>\*</sup>,  
T. Tarvainen<sup>‡</sup>, M. Vauhkonen<sup>‡</sup> and A. Zacharopoulos<sup>\*</sup>

June 7, 2007

## Abstract

In this study we consider the reconstruction of the smooth subdomain boundaries of piecewise constant coefficients of the Radiative Transfer Equation (RTE) from optical tomography data. The assumption is that the values of the absorption and scattering coefficients ( $\mu_a, \mu_{st}$ ) of the different subdomains are known *a priori* but the smooth subdomain boundaries where  $\mu_a$  and  $\mu_{st}$  are discontinuous are unknown. For the reconstruction of ( $\mu_a, \mu_{st}$ ) it is then sufficient to find the subdomain boundaries separating different values of the coefficients. This results to a nonlinear ill-posed inverse problem. In this study we propose a numerical algorithm for this inverse problem. The approach is based on the finite element discretization of the RTE. We formulate the forward problem as a mapping from a set of shape coefficients representing the shapes of the subdomain boundaries to optical tomography data, and derive the Jacobian of this forward mapping.

---

<sup>\*</sup>Department of Computer Science, University College London, Gower Street London, WC1E 6BT, UK

<sup>†</sup>Modeling and Numerical Simulation Group, Universidad Carlos III de Madrid, Avda de la Universidad 30, 28911 Leganes, Spain

<sup>‡</sup>Department of Physics, University of Kuopio, P.O.Box 1627, 70211 Kuopio, Finland

Then an iterative Newton type algorithm which seeks a boundary configuration minimizing the residual norm between measured and predicted data is implemented. The performance of the method is tested with simulated frequency domain optical tomography data from diffusive domains containing low-scattering (void) subdomains.

## References

- [1] , "S. R. Arridge and O. Dorn and J. P. Kaipio and V. Kolehmainen and M. Schweiger and T. Tarvainen and M. Vauhkonen and A. Zacharopoulos", "Reconstruction of Subdomain Boundaries of Piecewise Constant Coefficients of the Radiative Transport Equation from Optical Tomography Data", *Inverse Problems*, 22 (6), 2175–2198, 2006.
- [2] T. Tarvainen, M. Vauhkonen, V. Kolehmainen, S. R. Arridge, and J. P. Kaipio. Coupled radiative transfer equation and diffusion approximation model for photon migration in turbid medium with low-scattering and non-scattering regions. *Phys. Med. Biol.*, 50:4913–4930, 2005.
- [3] V. Kolehmainen, S. R. Arridge, W. R. B. Lionheart, M. Vauhkonen, and J. P. Kaipio. Recovery of region boundaries of piecewise constant coefficients of an elliptic PDE from boundary data. *Inverse Problems*, 15:1375–1391, 1999.
- [4] V. Kolehmainen, S. R. Arridge, M. Vauhkonen, and J. P. Kaipio. Simultaneous reconstruction of internal tissue region boundaries and coefficients in optical diffusion tomography. *Phys. Med. Biol.*, 45(11):3267–3283, 2000.